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Barrier Option Pricing of Exponential Ornstein-Uhlenbeck Model in Uncertain Environment☆

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Abstract

Barrier options are path-dependent options, and their return depends not only on the price of the underlying asset on the expiry date but also on whether the underlying asset reaches the prescribed barrier level during the contract's validity period. This paper mainly studies the barrier option pricing problem under the Ornstein–Uhlenbeck equation model under an uncertain environment. Assuming that the stock price obeys the Ornstein–Uhlenbeck equation model, the pricing formulas of four European barrier options are derived. Finally, several numerical examples are used to verify the effectiveness of the model.

Keywords: Uncertainty theory, Uncertain differential equation, Exponential OU model, Barrier option

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1. Introduction

Options are one of the most popular financial derivatives in the market, and they have good risk aversion functions. Barrier options are subject to certain restrictions during the effective process of options, and their purpose is to control investors' gains or losses within a specific range. A barrier option is a kind of option related to the path. In addition to its return depends on the underlying asset price at the expiration of the option; it is also associated with the underlying asset price reaching a certain level during the entire option period. The controllable effect of barrier options on the maximum potential return makes barrier options low in cost, which is quite popular among investors.

Barrier options are divided into two categories: knock-out options and knock-in options. Knock-out option refers to an option whose contract becomes invalid when the underlying asset price hits an obstacle. If the price of the underlying asset during the validity period of the option is greater than the barrier value, it is called a knock-down option; if the price of the underlying asset during the validity period of the option is less than the barrier value, it is called a knock-up option. The knock-in option refers to the option that the contract takes effect when the underlying asset price hits an obstacle. It is also divided into knock-down options and knock-up options. Since each type of option can be divided into the call and put options, we can divide barrier options into eight categories: down-and-in call options, down-and-in put options, up-and-in call options, up-and-out put options, down-and-out call options, down-and-out put options, up-and-out call options, and up-and-in put options.

Black and Scholes (1973) first published an article about barrier options and gave the pricing formula of down-and-in call options. After that, Reiner and Rubinstein (1991) added the pricing formulas of other types of European barrier options. Heynen and Kat (1994), Carr (1995) studied the rainbow barrier period of some barrier options. Li (2016) used no-arbitrage pricing and risk-neutral pricing principles to combine bounded differences with equations. And under the condition of transaction costs, the barrier option of the stochastic volatility model is priced, and the numerical solution of the option price is obtained. Yang (2017) combined the pricing problem of barrier options with the weighted average index jump-diffusion model, combined the double La-place transform method with the up-and-in call barrier option, and then used the Euler method and Monte-Carlo simulation method to simulate its numerical value. Xue and Deng (2018) combined the discrete barrier option with the Bates model and obtained a closed solution for the discrete barrier option price. They used the Girsanov's theorem, Fourier transform, and other methods and also performed numerical simulations.

However, many phenomena in the real world are neither completely random nor completely vague. Liu (2007) proposed uncertainty theory based on normality, duality, subadditivity, and product axioms to deal with this complex uncertainty. Liu (2019) further defined the uncertain process and uncertain differential equations, proposed the basic model of uncertain financial markets, and gave the pricing formulas of European call and put options under uncertain environments. Chen (2011) studied the pricing of American options under Liu's model and gave an analytical solution to the price of American options. Peng and Yao (2011) proposed a new stock model, Peng-Yao's model, and gave the pricing formulas of European and American options. Chen et al. (2013) proposed a stock model with periodic dividends. Zhang and Liu (2014) obtained the pricing formula of geometric average Asian options based on the uncertain stock model. Yao (2015) deduced the necessary and sufficient conditions for no arbitrage in the stock model. In the same year, Yao (2015) proposed an uncertain stock model with floating interest rates. Sun and Chen (2015) obtained the pricing formula of arithmetic average Asian options based on the uncertain stock model. Gao et al. (2017) received a lookback option pricing formula with a fixed strike price under the uncertain Ornstein–Uhlenbeck (OU) model.

This paper proposes the barrier option pricing of the exponential OU model under an uncertain environment. The rest of this article is as follows. The second section reviews some basic concepts in the uncertainty theory and introduces the uncertain exponential OU model. The third section introduces knock-in barrier options, considers European up-and-in call options and European down-and-in put options, and derives their option pricing formulas. The fourth section presents knock-out barrier options and considers European knock-up put options and European knock-down call options, and derives their option pricing formulas. Section 5 uses RuiSheng Technology's data to estimate the parameters of the pricing formulas, and calculates the price of four types of barrier options according to the algorithm. Finally, Section 6 gives some conclusions.

2. Preliminaries

Uncertainty theory was founded by Liu and refined by Liu (2009). This section will introduce some basic definitions and results in uncertainty theory.

2.1 Uncertain differential equation

Definition 2.1 (Liu (2009)) Let L be a σ-algebra on a nonempty set Γ. A set function M: $L \to [0, 1]$ is called an uncertain measure if it satisfies the following axioms.

Axiom 1 (Normality Axiom) $M\{\Gamma\}=1$ for the universal set Γ.

Axiom 2 (Duality Axiom) $M\{\Lambda\}+M\{\Lambda^c\}=1$ for any even Λ.

Axiom 3 (Subadditivity Axiom) For every countable sequence of events $\Lambda_1, \Lambda_2, \cdots$, we have

$$M\left\{\bigcup_{i=1}^{\infty}\Lambda_i\right\} \le \sum_{i=1}^{\infty} M\{\Lambda_i\}.$$

Axiom 4 (Product Axiom) Let (Γ_k, L_k, M_k) be unccertainty spaces for $k = 1, 2, \cdots$. The product uncertain measure M is an uncertain measure satisfying

$$M\left\{\prod_{k=1}^{\infty}\Lambda_k\right\} = \min_{k\ge 1} M_k\{\Lambda_k\}$$

where Λ_k are arbitrarily chosen events from L_k for k=1, 2, $\cdots$, respectively.

2.2 Uncertain exponential OU model

This section will introduce the exponential OU model for uncertain financial market. Let X_t be the stock price and Y_t be the bond price. Assuming that the stock price X_t follows a geometric Liu process. Then Liu's stock model is written as follows,

$$\begin{cases} dX_t = \mu X_t dt + \sigma X_t dC_t \\ dY_t = rY_t dt \end{cases} \tag{1}$$

where r is the riskless interest rate, μ is the stock drift, σ is the stock diffusion, and C_t is a Liu process. This model represents that the stocks have constant expected rate of return.

In order to reflect the actual situation of stock price changes better, this article considers the stock price obeys the uncertain exponential OU model.

The uncertain exponential OU model is written as follows

$$dX_t = \mu(1 - c\ln X_t)X_t dt + \sigma X_t dC_t \tag{2}$$

where $c > 0$, $\sigma > 0$, and μ are constants.

Theorem 2.4 (Sun et al. (2015)) Suppose that the stock price follows the model (2), then

$$X_t = \exp\left(\frac{1}{c} + \left(\ln X_0 - \frac{1}{c}\right)\exp(-\mu ct) + \sigma\int_0^t \exp\left(c\mu(s-t)\right)dC_t\right).$$

Theorem 2.5 (Dai et al. (2017)) Suppose that the stock price follows the model (2), then the inverse uncertainty distribution of X_t is

$$\Phi_t^{-1}(\alpha) = \exp\left(\exp(-\mu ct)\ln X_0 + (1-\exp(-\mu ct))\left(\frac{1}{c} + \frac{\sqrt{3}\sigma}{\mu c\pi}\ln\frac{\alpha}{1-\alpha}\right)\right).$$

3. Knock-in Options

This section will introduce the pricing formulas of the up-and-in call option and down-and- in put option. The up-and-in call option is a kind of barrier option that takes effect only when the spot price, which is below barrier level at first reaches the barrier level before the expiration date. The down-and-in put option is a kind of barrier option that takes effect only when the spot price, which is above barrier level at first goes down before the expiration date.

For simplicity, we define an indicator function

$$I_L(x) = \begin{cases} 1, & if\ x \ge L \\ 0, & if\ x < L \end{cases}$$

where L is a given real number.

3.1 European up-and-in call option

Consider the European up-and-in call option with a strike price K, an expiration date T and a barrier level L for some stock in the uncertain market. Let an uncertain process X_t denote the stock price and r denote the riskless interest rate. Then the European up-and-in call option price is

$$f_{ui}^{c}=\exp(-rT)E\left[I_L\left(\sup_{0\le t\le T}X_t\right)\cdot(X_T-K)^{+}\right].$$

Theorem 3.1. Suppose that a European up-and-in call option for the stock model (2) has a strike price is K, an expiration time T, and a barrier level L. Then the European up-and-in call option pricing formula is

$$f_{ui}^{c}=\exp(-rT)\int_{\beta}^{1}\left(\exp\left(\exp(-\mu cT)\ln X_0+(1-\exp(-\mu cT))\left(\frac{1}{c}+\frac{\sqrt{3}\sigma}{\mu c\pi}\ln\frac{\alpha}{1-\alpha}\right)\right)-K\right)^{+}d\alpha$$

where β is the infimum value of α, which satisfies the inequality

$$\exp\left(\exp(-\mu cT)\ln X_0+(1-\exp(-\mu cT))\left(\frac{1}{c}+\frac{\sqrt{3}\sigma}{\mu c\pi}\ln\frac{\alpha}{1-\alpha}\right)\right)\ge L.$$

Proof. According to Tian et al. (2019), the uncertain variable

$$I_L\left(\sup_{0\le t\le T}X_t\right)\cdot(X_T-K)^{+}$$

has an inverse uncertainty distribution

$$I_L\left(\sup_{0\le t\le T}X_t^{\alpha}\right)\cdot(X_T^{\alpha}-K)^{+}$$

where

$$X_t^{\alpha}=\exp\left(\exp(-\mu ct)\ln X_0+(1-\exp(-\mu ct))\left(\frac{1}{c}+\frac{\sqrt{3}\sigma}{\mu c\pi}\ln\frac{\alpha}{1-\alpha}\right)\right)$$

is the α-path of X_t. We have

$$\begin{aligned}f_{ui}^{c}&=\exp(-rT)\int_0^1 I_L\left(\sup_{0\le t\le T}X_t\right)\cdot(X_T-K)^{+}d\alpha\\&=\exp(-rT)\int_0^1 I_L\left(\sup_{0\le t\le T}X_t^{\alpha}\right)\cdot(X_T^{\alpha}-K)^{+}d\alpha,\end{aligned}$$

according to the expected value formula of uncertain variable. Besides, we have

$$I_L\left(\sup_{0\le t\le T}X_t^{\alpha}\right)=1$$

if and only if

$$\sup_{0\le t\le T}X_t^{\alpha}=\sup_{0\le t\le T}\left(\exp\left(\exp(-\mu ct)\ln X_0+(1-\exp(-\mu ct))\left(\frac{1}{c}+\frac{\sqrt{3}\sigma}{\mu c\pi}\ln\frac{\alpha}{1-\alpha}\right)\right)\right)\ge L.$$

Because $X_0< L$, we can replace the above inequality with inequality

$$\exp\left(\exp(-\mu cT)\ln X_0+(1-\exp(-\mu cT))\left(\frac{1}{c}+\frac{\sqrt{3}\sigma}{\mu c\pi}\ln\frac{\alpha}{1-\alpha}\right)\right)\ge L.$$

Thus, α need to satisfy the above inequality, and we make β be α's infimum value, therefore the price of European up-and-in call option is

$$\begin{aligned}f_{ui}^{c}&=\exp(-rT)\int_0^1 I_L\left(\sup_{0\le t\le T}X_t^{\alpha}\right)\cdot(X_T^{\alpha}-K)^{+}d\alpha\\&=\exp(-rT)\int_{\beta}^{1}(X_T^{\alpha}-K)^{+}d\alpha\\&=\exp(-rT)\int_{\beta}^{1}\left(\exp\left(\exp(-\mu cT)\ln X_0+(1-\exp(-\mu cT))\left(\frac{1}{c}+\frac{\sqrt{3}\sigma}{\mu c\pi}\ln\frac{\alpha}{1-\alpha}\right)\right)-K\right)^{+}d\alpha.\end{aligned}$$

The theorem is thus proved.

3.2 European down-and-in put option

Let K be the strike price of a European down-and-in put option, and its expiration date is T, the barrier level is L for some stock in the uncertain market. The stock price is represented by an uncertain process X_t and the riskless interest rate is represented by r. Then the option price is

$$f_{di}^{p}=\exp(-rT)E\left[\left(1-I_L\left(\inf_{0\le t\le T}X_t\right)\right)\cdot(K-X_T)^+\right].$$

Theorem 3.2. Consider that the strike price of an European down-and-in put option for stock model (2) is K, its expiration time is T, and its barrier level is L. Then the pricing formula of the European up-and-in put option is

$$f_{di}^{p}=\exp(-rT)\int_0^{\beta}\left(K-\exp\left(\exp(-\mu cT)\ln X_0+(1-\exp(-\mu cT))\left(\frac{1}{c}+\frac{\sqrt{3}\sigma}{\mu c\pi}\ln\frac{\alpha}{1-\alpha}\right)\right)\right)^+d\alpha$$

where β is the supremum value of α, which satisfies the inequality

$$\exp\left(\exp(-\mu cT)\ln X_0+(1-\exp(-\mu cT))\left(\frac{1}{c}+\frac{\sqrt{3}\sigma}{\mu c\pi}\ln\frac{\alpha}{1-\alpha}\right)\right)<L.$$

Proof. According to Tian et al. (2019), the uncertain variable

$$\left(1-I_L\left(\inf_{0\le t\le T}X_t\right)\right)\cdot(K-X_T)^+$$

has an inverse uncertainty distribution

$$\left(1-I_L\left(\inf_{0\le t\le T}X_t^{1-\alpha}\right)\right)\cdot(K-X_T^{1-\alpha})^+$$

where

$$X_t^{\alpha}=\exp\left(\exp(-\mu ct)\ln X_0+(1-\exp(-\mu ct))\left(\frac{1}{c}+\frac{\sqrt{3}\sigma}{\mu c\pi}\ln\frac{\alpha}{1-\alpha}\right)\right)$$

is the α-path of X_t. We have

$$\begin{aligned}f_{di}^{p}&=\exp(-rT)\int_0^1\left(1-I_L\left(\inf_{0\le t\le T}X_t^{1-\alpha}\right)\right)\cdot(K-X_T^{1-\alpha})^+d\alpha\\&=\exp(-rT)\int_0^1\left(1-I_L\left(\inf_{0\le t\le T}X_t^{\alpha}\right)\right)\cdot(K-X_T^{\alpha})^+d\alpha,\end{aligned}$$

according to the expected value formula of uncertain variable. Besides, we have

$$I_L\left(\inf_{0\le t\le T}X_t^{\alpha}\right)=0$$

if and only if

$$\inf_{0\le t\le T}X_t^{\alpha}=\inf_{0\le t\le T}\left(\exp\left(\exp(-\mu ct)\ln X_0+(1-\exp(-\mu ct))\left(\frac{1}{c}+\frac{\sqrt{3}\sigma}{\mu c\pi}\ln\frac{\alpha}{1-\alpha}\right)\right)\right)<L.$$

Because $X_0>L$, we can replace the above inequality with inequality

$$\exp\left(\exp(-\mu cT)\ln X_0+(1-\exp(-\mu cT))\left(\frac{1}{c}+\frac{\sqrt{3}\sigma}{\mu c\pi}\ln\frac{\alpha}{1-\alpha}\right)\right)<L.$$

Thus, α need to satisfy the above inequality, and we make β be α's supremum value, therefore the price of European down-and-in put option is

$$\begin{aligned}f_{di}^{p}&=\exp(-rT)\int_0^1\left(1-I_L\left(\inf_{0\le t\le T}X_t^{\alpha}\right)\right)\cdot(K-X_T^{\alpha})^+d\alpha\\&=\exp(-rT)\int_0^{\beta}(K-X_T^{\alpha})^+d\alpha\\&=\exp(-rT)\int_0^{\beta}\left(K-\exp\left(\exp(-\mu cT)\ln X_0+(1-\exp(-\mu cT))\left(\frac{1}{c}+\frac{\sqrt{3}\sigma}{\mu c\pi}\ln\frac{\alpha}{1-\alpha}\right)\right)\right)^+d\alpha.\end{aligned}$$

The theorem is thus proved.

4. Knock-out Options

This section will introduce the pricing formulas of European up-and-out put option and European down-and-out call option.

4.1 European up-and-out put option

Let K be the strike price of a European up-and-out put option, and its expiration date is T, the barrier level is L for some stock in the uncertain market. The stock price is represented by an uncertain process X_t and the riskless interest rate is represented by r. Then the option price is

$$f_{do}^{p}=\exp(-rT)E\left[\left(1-I_L\left(\sup_{0\le t\le T}X_t\right)\right)\cdot(K-X_T)^+\right].$$

Theorem 4.1. Consider that the strike price of a European up-and-out put option for stock model (2) is K, its expiration time is T, and its barrier level is L. Then the pricing formula of the European up-and-out put option is

$$f_{uo}^{p} = \exp(-rT)\int_0^{\beta}\left(K - \exp\left(\exp(-\mu cT)\ln X_0 + (1-\exp(-\mu cT))\left(\frac{1}{c} + \frac{\sqrt{3}\sigma}{\mu c\pi}\ln\frac{\alpha}{1-\alpha} \right) \right) \right)^{+} d\alpha$$

where β is the supremum value of α, which satisfies the inequality

$$\exp\left(\exp(-\mu cT)\ln X_0 + (1-\exp(-\mu cT))\left(\frac{1}{c} + \frac{\sqrt{3}\sigma}{\mu c\pi}\ln\frac{\alpha}{1-\alpha} \right) \right) < L.$$

Proof. According to Tian et al. (2019), the uncertain variable

$$\left(1 - I_L\left(\sup_{0\le t\le T} X_t\right)\right)\cdot(K - X_T)^{+}$$

has an inverse uncertainty distribution

$$\left(1 - I_L\left(\sup_{0\le t\le T} X_t^{1-\alpha}\right)\right)\cdot(K - X_T^{1-\alpha})^{+}$$

where

$$X_t^{\alpha} = \exp\left(\exp(-\mu ct)\ln X_0 + (1-\exp(-\mu ct))\left(\frac{1}{c} + \frac{\sqrt{3}\sigma}{\mu c\pi}\ln\frac{\alpha}{1-\alpha} \right) \right)$$

is the α-path of X_t. We have

$$\begin{aligned} f_{uo}^{p} &= \exp(-rT)\int_0^1\left(1 - I_L\left(\sup_{0\le t\le T} X_t^{1-\alpha}\right)\right)\cdot(K - X_T^{1-\alpha})^{+}d\alpha \\ &= \exp(-rT)\int_0^1\left(1 - I_L\left(\sup_{0\le t\le T} X_t^{\alpha}\right)\right)\cdot(K - X_T^{\alpha})^{+}d\alpha, \end{aligned}$$

according to the expected value formula of uncertain variable. Besides, we have

$$I_L\left(\sup_{0\le t\le T} X_t^{\alpha}\right) = 0$$

if and only if

$$\sup_{0\le t\le T} X_t^{\alpha} = \sup_{0\le t\le T}\left(\exp\left(\exp(-\mu ct)\ln X_0 + (1-\exp(-\mu ct))\left(\frac{1}{c} + \frac{\sqrt{3}\sigma}{\mu c\pi}\ln\frac{\alpha}{1-\alpha} \right) \right) \right) < L.$$

Because $X_0 < L$, we can replace the above inequality with inequality

$$\exp\left(\exp(-\mu cT)\ln X_0 + (1-\exp(-\mu cT))\left(\frac{1}{c} + \frac{\sqrt{3}\sigma}{\mu c\pi}\ln\frac{\alpha}{1-\alpha} \right) \right) < L.$$

Thus, α need to satisfy the above inequality, and we make β be α's supremum value, therefore the price of European up-and-out put option is

$$\begin{aligned} f_{uo}^{p} &= \exp(-rT)\int_0^1\left(1 - I_L\left(\sup_{0\le t\le T} X_t^{\alpha}\right)\right)\cdot(K - X_T^{\alpha})^{+}d\alpha \\ &= \exp(-rT)\int_0^{\beta}(K - X_T^{\alpha})^{+}d\alpha \\ &= \exp(-rT)\int_0^{\beta}\left(K - \exp\left(\exp(-\mu cT)\ln X_0 + (1-\exp(-\mu cT))\left(\frac{1}{c} + \frac{\sqrt{3}\sigma}{\mu c\pi}\ln\frac{\alpha}{1-\alpha} \right) \right) \right)^{+} d\alpha. \end{aligned}$$

The theorem is thus proved.

4.2 European down-and-out call option

Let K be the strike price of a European down-and-out call option, and its expiration date is T, the barrier level is L for some stock in the uncertain market. The stock price is represented by an uncertain process X_t and the riskless interest rate is represented by r. Then the option price is

$$f_{do}^{c} = \exp(-rT)E\left[I_L\left(\inf_{0\le t\le T} X_t\right)\cdot(X_T - K)^{+} \right].$$

Theorem 4.2. Consider that the strike price of a European down-and-out call option for stock model (2) is K, its expiration time is T, and its barrier level is L. Then the pricing formula of the European down-and-out call option is

$$f_{do}^{c} = \exp(-rT)\int_{\beta}^{1}\left(\exp\left(\exp(-\mu cT)\ln X_0 + (1-\exp(-\mu cT))\left(\frac{1}{c} + \frac{\sqrt{3}\sigma}{\mu c\pi}\ln\frac{\alpha}{1-\alpha} \right) \right) - K \right)^{+} d\alpha$$

where β is the infimum value of α, which satisfies the inequality

$$\exp\left(\exp(-\mu cT)\ln X_0 + (1-\exp(-\mu cT))\left(\frac{1}{c}+\frac{\sqrt{3}\sigma}{\mu c\pi}\ln\frac{\alpha}{1-\alpha}\right)\right) \geq L.$$

Proof. According to Tian et al. (2019), the uncertain variable

$$I_L\left(\inf_{0\le t\le T} X_t\right)\cdot(X_T-K)^+$$

has an inverse uncertainty distribution

$$I_L\left(\inf_{o\le t\le T} X_t^\alpha\right)\cdot(X_T^\alpha-K)^+$$

where

$$X_t^\alpha = \exp\left(\exp(-\mu ct)\ln X_0 + (1-\exp(-\mu ct))\left(\frac{1}{c}+\frac{\sqrt{3}\sigma}{\mu c\pi}\ln\frac{\alpha}{1-\alpha}\right)\right)$$

is the α-path of X_t. We have

$$\begin{aligned} f_{do}^c &= \exp(-rT)\int_0^1 I_L\left(\inf_{0\le t\le T} X_t\right)\cdot(X_T-K)^+ d\alpha \\ &= \exp(-rT)\int_0^1 I_L\left(\inf_{0\le t\le T} X_t^\alpha\right)\cdot(X_T^\alpha-K)^+ d\alpha, \end{aligned}$$

according to the expected value formula of uncertain variable. Besides, we have

$$I_L\left(\inf_{0\le t\le T} X_t^\alpha\right)=1$$

if and only if

$$\inf_{0\le t\le T} X_t^\alpha = \inf_{0\le t\le T}\left(\exp\left(\exp(-\mu ct)\ln X_0 + (1-\exp(-\mu ct))\left(\frac{1}{c}+\frac{\sqrt{3}\sigma}{\mu c\pi}\ln\frac{\alpha}{1-\alpha}\right)\right)\right) \geq L.$$

Because $X_0 > L$, we can replace the above inequality with inequality

$$\exp\left(\exp(-\mu cT)\ln X_0 + (1-\exp(-\mu cT))\left(\frac{1}{c}+\frac{\sqrt{3}\sigma}{\mu c\pi}\ln\frac{\alpha}{1-\alpha}\right)\right) \geq L.$$

Thus, α need to satisfy the above inequality, and we make β be α's infimum value, therefore the price of European down-and-out call option is

$$\begin{aligned} f_{do}^c &= \exp(-rT)\int_0^1 I_L\left(\inf_{0\le t\le T} X_t^\alpha\right)\cdot(X_T^\alpha-K)^+ d\alpha \\ &= \exp(-rT)\int_\beta^1 (X_T^\alpha-K)d\alpha \\ &= \exp(-rT)\int_\beta^1\left(\exp\left(\exp(-\mu cT)\ln X_0 + (1-\exp(-\mu cT))\left(\frac{1}{c}+\frac{\sqrt{3}\sigma}{\mu c\pi}\ln\frac{\alpha}{1-\alpha}\right)\right) - K\right)^+ d\alpha. \end{aligned}$$

The theorem is thus proved.

5. Simulation example

5.1 Parameter estimation

Assuming that the sample data at times $t_1, t_2, \cdots, t_n$ are $X_{t_1}, X_{t_2}, \cdots, X_{t_n}$, respectively. The given confidence level is θ. According to Yang et al. (2020), we use the following method to estimate the parameters (μ, c, σ),

$$\begin{cases} \min\limits_{\mu,c,\sigma} X_{t_n}^\theta - X_{t_n}^{1-\theta} \\ s.t.\ X_{t_i}^{1-\theta} \le X_{t_i} \le X_{t_i}^\theta, i=1,2,\cdots,n \\ where\ X_{t_i}^\theta = \exp\left(\exp(-\mu ct_i)\ln X_0 + (1-\exp(-\mu ct_i))\left(\frac{1}{c}+\frac{\sqrt{3}\sigma}{\mu c\pi}\ln\frac{\alpha}{1-\alpha}\right)\right). \end{cases} \tag{3}$$

In this inequality group, X_{t_n} and $X_{t_n}^{1-\theta}$ reached the minimum value of the difference at time t_n. Use $\left(\mu^*, c^*, \sigma^*\right)$ to represent the optimal solution of the minimization problem (3). According to Yang et al. (2020), the estimation $\left(\mu^*, c^*, \sigma^*\right)$ of equation (2) is the optimal solution of the minimization problem (3).

Table 1: Sample data of RuiSheng Technology

X_1 43.15	X_7 49.45	X_{13} 57.50
X_2 42.75	X_8 48.85	X_{14} 56.85
X_3 43.60	X_9 47.80	X_{15} 59.30
X_4 46.45	X_{10} 47.55	X_{16} 61.30
X_5 47.35	X_{11} 49.10	X_{17} 59.80
X_6 46.90	X_{12} 52.35	X_{18} 60.85

We choose RuiSheng Technology as the research object, we have selected the closing price from June 16th , 2020 to July 13th, 2020, and the data is displayed in the Table. There are 18 days of total trading days, i.e., n = 18. For the uncertain exponential OU process, taking $\theta = 0.90$ and solving the minimization problem (4), we obtain the estimation

$$\left(\mu^*, c^*, \sigma^*\right) = \left(0.0317, 0.1234, 0.0078\right),$$

then, the uncertain exponential OU process is

$$dX_t = 0.0317(1-0.1234\ln X_t)X_t dt + 0.0078 X_t dC_t.$$

5.2 European up-and-in call option

Assume the daily interest rate is r = 0.0064%, and the stock has a spot price X_0=43.15 with the parameters c = 0.1234, μ = 0.0317 and σ = 0.0078. Then the price of a European up-and-in call option with a striking price K = 48, an expiration data T = 30 and a barrier lever L = 49 is

$$f_{ui}^c = 4.6065.$$

Then, we give the curve graphs of barrier option formula with different parameters as follows. Figure 1(a) denotes the price f_{ui}^c with the change of the riskless interest rate r, Figure 1(b) denotes the price f_{ui}^c with the change of strike price K; Figure 1(c) denotes the price f_{ui}^c with the change of spot price X_0; Figure 1(d) denotes the price with the change of barrier level L. If one parameter changes, other parameters remain unchanged.

5.3 European down-and-in put option

The price of a European down-and-in put option with a striking price K = 65, an expiration data T = 30 and a barrier lever L = 35 is

$$f_{di}^p = 3.3649.$$

Then, we give the curve graphs of barrier option formula with different parameters as follows. Figure 2(a) denotes the price f_{di}^p with the change of the riskless interest rate r; Figure 2(b) denotes the price f_{di}^p with the change of strike price K; Figure 2(c) denotes the price f_{di}^p with the change of spot price X_0; Figure 2(d) denotes the price f_{di}^p with the change of barrier level L. If one parameter changes, other parameters remain unchanged.

5.4 European up-and-out put option

The price of a European up-and-out put option with a striking price K = 65, an expiration data T = 30 and a barrier lever L = 50 is

$$f_{uo}^p = 1.6326.$$

Then, we give the curve graphs of barrier option formula with different parameters as follows. Figure 3(a) denotes the price f_{uo}^p with the change of the riskless interest rate r; Figure 3(b) denotes the price f_{uo}^p with the change of strike price K; Figure 3(c) denotes the price f_{uo}^p with the change of spot price X_0; Figure 3(d) denotes the price f_{uo}^p with the change of barrier level L. If one parameter changes, other parameters remain unchanged.

5.5 European down-and-out call option

The price of a European down-and-out call option with a striking price K = 40, an expiration data T = 30 and a barrier lever L = 37 is

$$f_{do}^c = 5.6500.$$

Then, we give the curve graphs of barrier option formula with different parameters as follows. Figure 4(a) denotes the price f_{do}^c with the change of the riskless interest rate r; Figure 4(b) denotes the price f_{do}^c with the change of strike price K; Figure 4(c) denotes the price f_{do}^c with the change of spot price X_0; Figure 4(d) denotes the price f_{do}^c with the change of barrier level L. If one parameter changes, other parameters remain unchanged.

6. Conclusions

This paper investigated the barrier option pricing problem of the exponential Ornstein–Uhlenbeck model in an uncertain environment. It studied and derived the pricing formulas of four options, which are up-and-in call option, down-and-in put option, up-and-out put option, down-and- out call option. And based on the stock price of Ruisheng Technology, the prices of these four barrier options are derived.

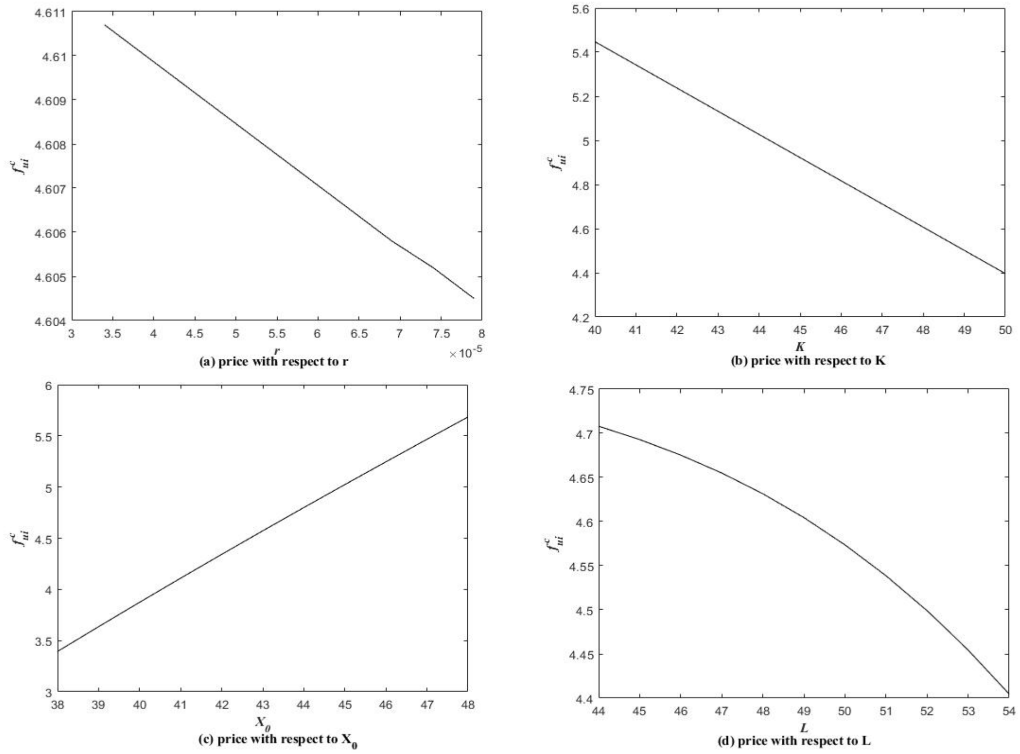

Figure 1: European up-and-in call option price f_{ui}^{c} with different parameters

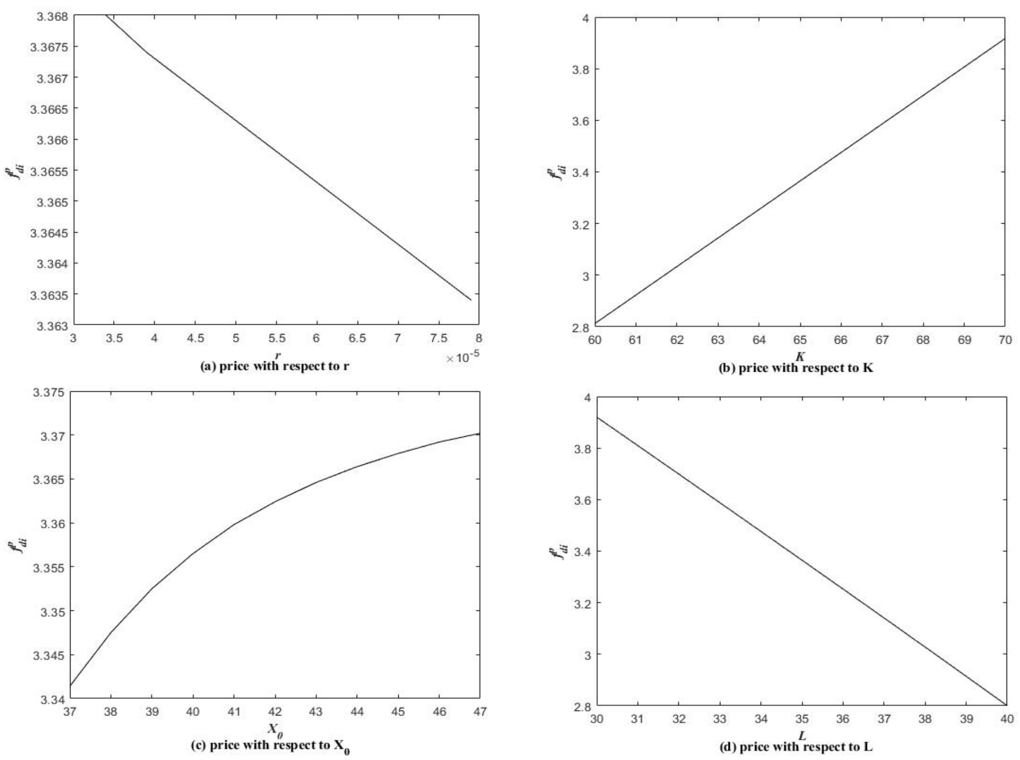

Figure 2: European down-and-in put option price f_{di}^{p} with different parameters

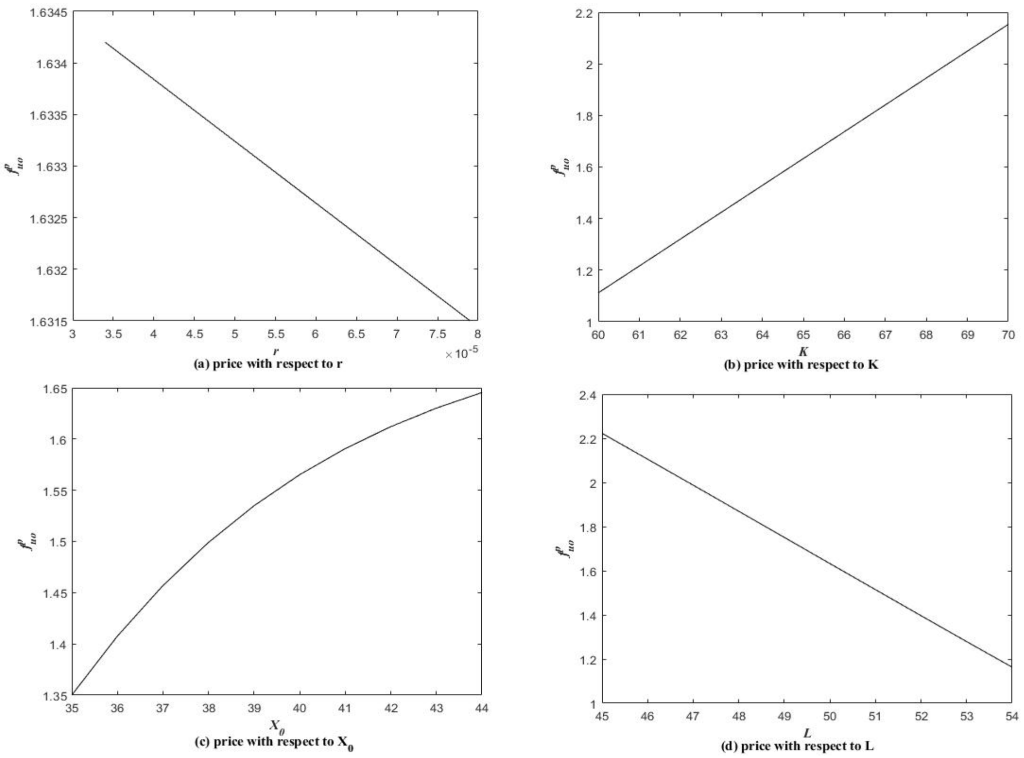

Figure 3: European up-and-out put option price f_{uo}^{p} with different parameters

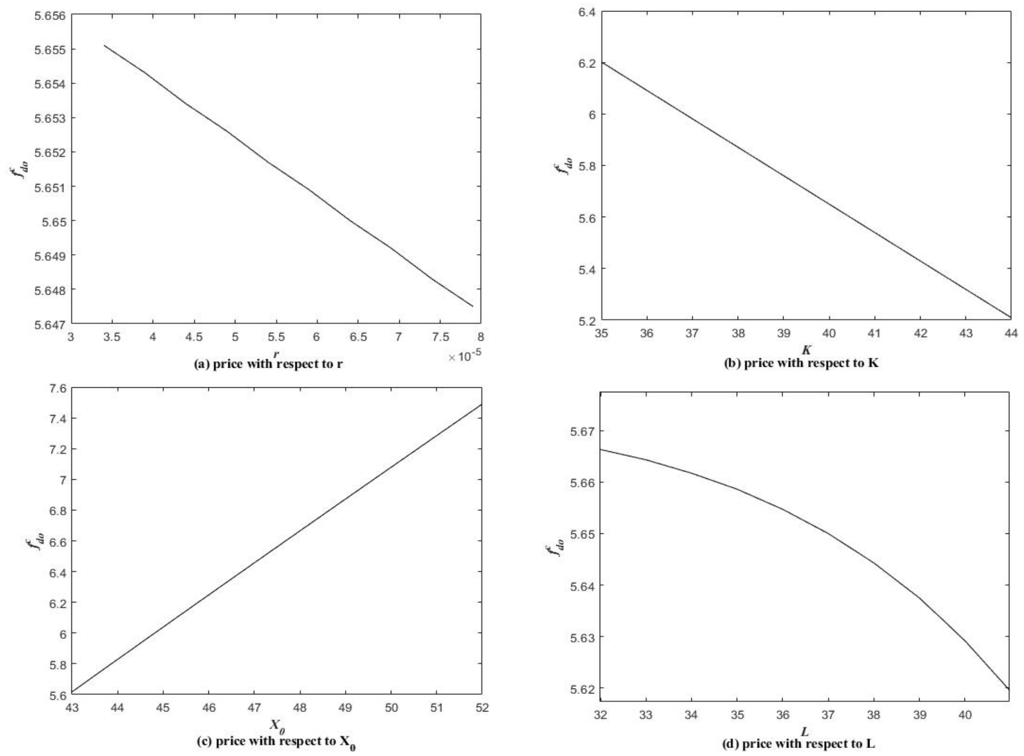

Figure 4: European down-and-out call option price f_{do}^{c} with different parameters

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