



Original article

An Advanced Celestial Method to Determine Ship's Position by Simultaneous Observation of Altitude and Azimuth of the Sun

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Abstract

A ship's position as determined by the observation of celestial bodies is a traditional method with important advantages, such as reliability, independence and a low cost. Global satellite navigation systems, with many outstanding advantages in terms of accuracy and continuity, have become the main method of ship positioning in offshore navigation. Ship positioning using celestial body observation is still a backup method in the event of unusual incidents. Currently, during the daytime, it is only possible to apply the celestial navigation method to determine the ship's position by observing the altitude of the sun. In order to reduce geometrical errors, this traditional method requires time for a certain change of the azimuth of the sun and therefore depends much on estimated errors and the effects of external conditions. Moreover, the basic requirement of the backup method is to provide a ship position quickly during offshore navigation, without the position being determined by a global satellite positioning system. To overcome the above limitations, the paper proposes a new approach to determine a ship's position by simultaneously observing the altitude and azimuth of the sun. A program for calculating the position of a ship with high reliability and applicability based on the new algorithm is also devised and shown to be highly effective in practice.

Keywords: Dead reckoning, estimated position, simultaneous observation, altitude and azimuth, celestial navigation fix

1. Introduction

Nowadays, by the rapid development of marine technology science, modern equipment has been invented to calculate ship's position accurately, effectively and quickly such as Global Positioning System (GPS). However, one of the important issues that arise for navigators in every voyage is the daily determination of the astronomical ship's position. Celestial ship positioning has been one of the traditional methods to determine the ship's position at sea. The key point for this is to observe the celestial bodies such as the sun, moon, planets and stars (Hsu, 2005). Ship's position determination by celestial body observation is being used as backup system due to developing technology in navigation devices but it is still an essential part of navigation to get a fix on ship's position.

Recently, some new methods have been proposed to solve the equations specifically for the ship's position have been proposed. The SEEM of Chen et al. (2003) requests a complicated procedure for determining ship's position. Tsou et al. (2012) used the evolution algorithms in artificial intelligence, applied the genetic algorithm and the particle swarm optimization to fix an optimal ship's position. A stand-alone celestial navigation positioning method was presented by Pierros (2018) for both stationary and moving observers. Lusic (2018) used the positioning equations through the spherical trigonometry. Although the above-mentioned researches have significantly contributed to the celestial positioning fixing, they are required to measure the altitude or azimuth of either at least two celestial bodies or a single celestial body at two different times. Moreover, the existing celestial methods of celestial navigation with more than two celestial bodies are time-consuming to conduct because many steps to process.

Ship's position determination by simultaneous observation of bearing and distance to a terrestrial object is a simple method with high accuracy and usually applied when coastal navigation. In celestial navigation, the azimuth measurement to a celestial body is complicated and inaccurate. Only the observation of the sun is more simple and accurate. The celestial ship positioning in the daytime is mainly non-simultaneous observation of the sun (Krasavtsev et al., 1970). This non-simultaneous method is inaccurate, needs to be conducted when the sun crosses the observer's meridian

and the time period with two attempts of azimuth measurement is great. This paper proposed an advance celestial method to determine ship's position by simultaneous observation of altitude and azimuth of the sun. This is a new celestial ship positioning. It can overcome the limitations of non-simultaneous method, quickly fix the ship's position in the daytime and is not influenced much by the error of the estimated position. Based on the methodology of the new method, the new algorithm is introduced and the program to automatically calculate the ship's position with high accuracy. This automatic program can improve the quality of the celestial ship positioning method, satisfy the requirements as a backup method and meet the standards for safe modern navigation (STCW 78/2010). Moreover, in comparison with observation of other celestial bodies, the sun is more convenient and easier to observe in daytime. The ship's officers are also familiar with the process of the sun observation for error determination of the compass. Therefore, it can reduce the error by unskilled operation during observation process.

The main contributions of this research are as follows:

First, our proposed method only need the sun to perform at a given time. Therefore, it can be useful in almost daytime where only the sun is an identified single celestial body.

Secondly, this method is not time-consuming and can be the reduce the error between observations. With its advantages such as low cost, high independence and the developments presented in this research, it can be used as backup system for GPS when this equipment has failure. This method can be a useful reference for navigators and marine students.

To verify the effectiveness of the proposed program, an experiment on board ship was performed. The results showed that this program has fast performance for accurately determining the ship's position. This article consists of the parts as follows: Traditional mathematical methodology for ship positioning is presented in Section 2. The detail of the proposed technique is shown in Section 3. Results of the experiment are described in Section 4. The conclusion is summarized in Section 5.

2. Ship’s Positioning by Non-simultaneously Observing the Sun

2.1. Methodology

The condition to measure altitude of a celestial body is simultaneous observation of this body and the true horizon. Therefore, the ship position fixing by celestial method should be conducted at nautical twilight. In the daytime, only the sun can be observed from a ship, so the non-simultaneous observation method is often used. Moreover, due to the daily motion, the position of the sun always changes from the observer's meridian and zenith. In order to reduce the geometrical error, the time of observations should be calculated so that the variation of the azimuth of the sun $\Delta A \geq 30^\circ$ (Bowditch, 2002).

Supposed that, at the first attempt of the sun’s altitude measurement, the circle of an equal altitude (COP) h_1 with the center as sub-stellar S_1 , radius as zenith distance $z_1 = 90^\circ - h_1$ is achieved as the line of position I-I (as can be seen in Figure 1). When the azimuth and altitude of the sun change, the sub-stellar moves along the arc S_1S_2 to point S_2 . The second measurement is conducted to get the second COP h_2 , and the corresponding LOP II-II. If the ship is not moving (mooring or anchoring), her position will be the intersection M_0 of two above LOP $M_0 = I-I \cap II-II$.

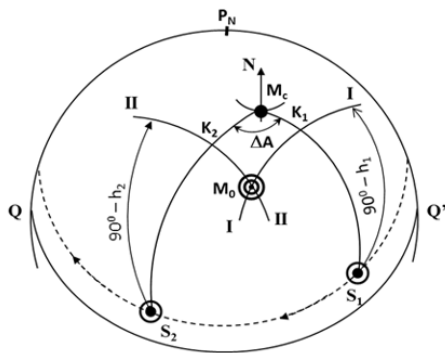


Figure 1: In case the ship is not moving

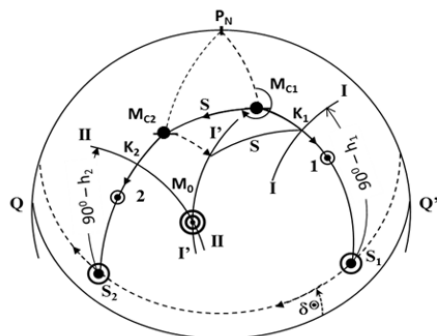


Figure 2: In case the ship is moving

When navigation, during the time between two

attempts of measurement, the ship is travelling a distance $M_{C1}M_{C2} = S$ (in Figure 2). At position M_{C1} , the altitude h_1 is measured and the LOP I-I is constructed based on the zenith of M_{C1} and the position of the sun S_1 (with the elements of triangle of estimated position $\varphi_{C1}, \lambda_{C1}, \delta_1, t_1$). Similarly, at position M_{C2} , the altitude h_2 is measured and the LOP II-II is constructed based on the zenith of M_{C2} and the position of the sun S_2 (with the elements of triangle of estimated position $\varphi_{C2}, \lambda_{C2}, \delta_2, t_2$).

In order to achieve ship’s position, two LOP I-I and II-II should be converted to a same moment. This can be done by Diagrammatic Method. Supposed that, the ship is travelling a distance S in the direction S_1S_2 , then the LOP I-I will move to $I'-I'$. In Figure 3, $I'-I'$ can be plotted from M_{C2} with the elements $(\Delta h_1 = h_{S1} - h_{C1}$ and $A_{C1})$. Therefore, the ship’s position at the second observation will be the intersection between II-II and the translation line $I'-I'$ ($M_0 = I'-I' \cap II-II$).

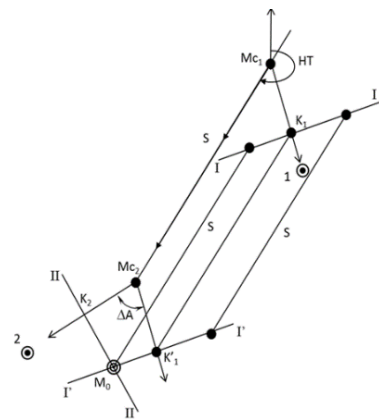


Figure 3: Altitude is converted to a same zenith

2.2. Ship’s Positioning by Non-simultaneously Observing the Sun Procedure

Observation moment selection:

According to Burch (2010), azimuth variation of a celestial body in observed daily motion is calculated as follows:

$$\Delta A = (\cos A \cos \varphi \tanh - \sin \varphi) \Delta t \tag{1}$$

Azimuth changes the most if it reaches the values $A = 0^\circ$ or 180° , when the celestial body crosses the observer’s meridian. Thus, the most appropriate moment for observation to fix the ship’s position is $2^h00^m - 2^h30^m$ before or after the sun crosses the observer’s upper meridian at mid latitude and $40^m - 1^h30^m$ at low latitude.

At first observation from estimated position $M_{C1}(\varphi_{C1}, \lambda_{C1})$, measure the sun’s altitude. Time, speed, course, air

pressure, temperature and the height of observer’s eye should be noted.

The measured altitude variation should be adjusted as follows:

$$\Delta h_{S1} = OC_{TB1} + i + s - d + \Delta h + \Delta h_{T_1 B_1} \quad (2)$$

With T_G , declination δ_1 and Greenwich hour angle t_{G1} of the sun can be calculated by nautical almanac. Local hour angle can be calculated as follows:

$$t_{L1} = t_{G1} \pm \lambda_W^E \quad (3)$$

By the parameters $(\varphi_{C1}, \delta_1, t_{L1})$, the altitude and azimuth of the sun (h_{C1}, A_{C1}) can be achieved by checking in specific mathematic table of calculating by general formulas.

Similarly, the second observation is carried out from the estimated position $M_{C2}(\varphi_{C2}, \lambda_{C2})$ with the azimuth variation is around $35^\circ - 40^\circ$, based on the real situation.

In nautical chart, the astronomical LOP can be plotted as in Figure 4:

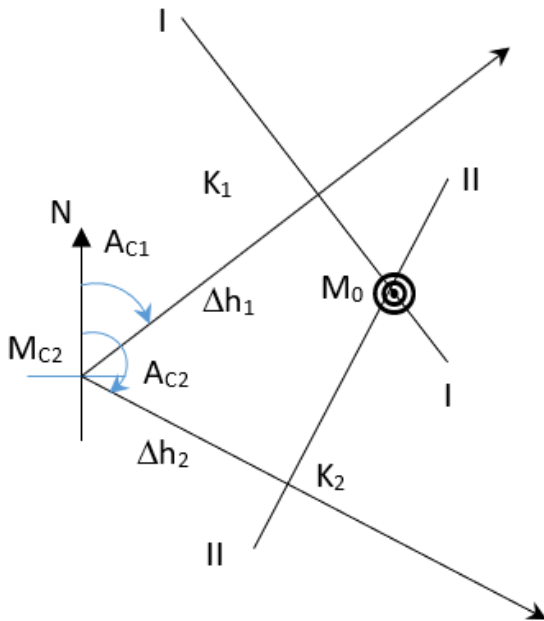


Figure 4: Ship’s positioning by non-simultaneously observing the sun procedure

LOP I-I: estimated position $M_{C1}(\varphi_{C1}, \lambda_{C1})$, calculated azimuth A_{C1} and altitude variation $\Delta h_1 = h_{S1} - h_{C1}$.

LOP II-II: estimated position $M_{C2}(\varphi_{C2}, \lambda_{C2})$, calculated azimuth A_{C2} and altitude variation $\Delta h_2 = h_{S2} - h_{C2}$.

3. Ship Positioning by Simultaneous Observation of Altitude and Azimuth of the Sun

Ship positioning by non-simultaneous observing the sun has some disadvantages. It should be carried out when the sun crosses the observer’s meridian. It is also effected by the error of estimated position, especially the error by the translation of LOP. In order to overcome these demerits, an advance method by simultaneous observation of altitude and azimuth of sun is proposed in this paper. The procedure of this method is as follows:

Step 1: Determine searching domain

The aim of this step is to determine the area that is adjacent to the estimated location with a 95% probability of containing the actual ship's position according to the standard of positioning accuracy (IMO, 1983). Searching area is determined as follows:

Latitude limitation: $\varphi_{min} - \varphi_{max}$ (Southern limitation $\varphi_0 = \varphi_{min}$ and Northern limitation $\varphi_a = \varphi_{max}$)

Longitude limitation: $\lambda_{min} - \lambda_{max}$ (Western limitation $\lambda_0 = \lambda_{min}$ and Eastern limitation $\lambda_b = \lambda_{max}$)

with:

$\varphi_{min} = \varphi_C - |\Delta\varphi_C|$, $\varphi_{max} = \varphi_C + |\Delta\varphi_C|$, where $\Delta\varphi_C$ is the error of the estimated latitude φ_C .

$\lambda_{min} = \lambda_C - |\Delta\lambda_C|$, $\lambda_{max} = \lambda_C + |\Delta\lambda_C|$, where $\Delta\lambda_C$ is the error of the estimated longitude λ_C .

The error of the estimated position is calculated based on the value of mean square error (R) of ship’s position ($|\Delta\varphi_C| = kR$, $|\Delta\lambda_C| = R$).

where:

k is coefficient of scale increase along meridian or the increase of Meridional Part in Mercator chart (Nguyen, 2019).

R is radius of the probability area containing the ship's location, is calculated such that the probability that the actual ship’s position is in the searching domain is greater than 95%.

Actually, in some special circumstances when the celestial method is used as the backup to determine the ship’s position, there is only estimated position of a ship. Then, according to Dmitriev (2009), the radius of mean square error (R) is calculated as follows [12]:

$$R = \sqrt{(S_{TK} \varepsilon_L)^2 + (S_{TK} \varepsilon_{TK})^2} \quad (4)$$

where:

S_{TK} is the distance the ship travelled.

ε_L is the error in compass adjustment.

ε_{TK} is the error in tachometer adjustment.

Supposed that, according to the tachometer, the ship travelled a distance as 100 nm, the error in tachometer adjustment is 0.6%, the error in compass adjustment is 005. The radius of mean square error (R) is:

$$R = \sqrt{\left(100 \frac{0.5}{57^{\circ}3}\right)^2 + \left(100 \frac{0.6}{100}\right)^2} = 1.06nm$$

To calculate the estimated error, the error radius of error is determined as $3R = 3.18nm$ (99.7%).

Step 2: Establish the set of assumed ship’s positions in searching domain

Construct the longitude – latitude net to establish the set of points $A = \{M_{xy}(\varphi_x, \lambda_y)\}$ as can be seen in Figure 5, where:

$$x = \{1, 2, \dots, b\} \text{ and } y = \{1, 2, \dots, a\}$$

Southern latitude limitation is $\varphi_0 = \varphi_{\min}$ and Northern latitude limitation is $\varphi_a = \varphi_{\max}$.

Western longitude limitation is $\lambda_0 = \lambda_{\min}$ and Eastern longitude limitation is $\lambda_b = \lambda_{\max}$.

The guaranteed difference of longitude and latitude: $\varphi_{i+1} - \varphi_i = 0^{\circ}000001$ and $\lambda_{i+1} - \lambda_i = 0^{\circ}000001$.

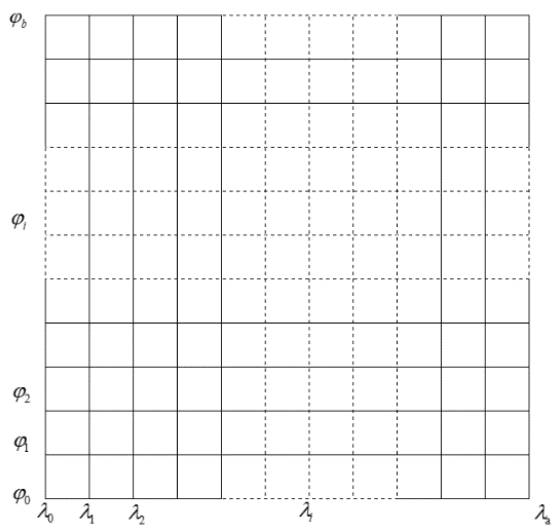


Figure 5: The set of points in searching domain

On board, the sun observation is conducted and the altitude and azimuth of the sun after adjustment is (h_s, A_T) . The ship positioning operation in Mercator chart is that the circle of equal altitude (with the center as sub-stellar $S(\varphi_s, \lambda_s)$, the radius as true distance (h_s) intersects

with measured azimuth (A_T) at ship’s position F. However, the true position F is only assumed position because the the LOP (h_s, h_s') cannot be plotted on chart due to the distance h_s is too great.

Calculate the distance from a random point $M_{xy} \in \{A\}$ to F, because the distance FS is too great, then the angle $M_{xy}EF$ is considered as 90° (in Figure 6).

In right triangle $M_{xy}EF$:

$$(M_{xy}F)^2 = (M_{xy}E)^2 + (EF)^2 \tag{5}$$

where:

$$EF \approx \widehat{EF} = FS \times \alpha = h_s \times \alpha \tag{6}$$

$$\alpha = A_T - K \tag{7}$$

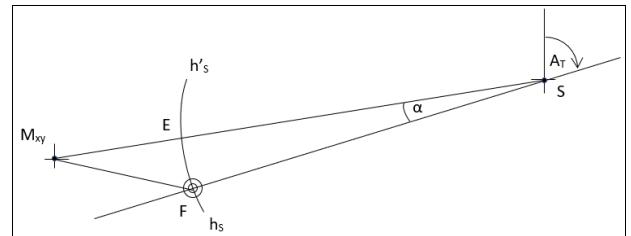


Figure 6: Determine the most probable position

K is the direction from M_{xy} to S in Mercator chart and calculated according to Frost (2011) as follows:

$$K = \tan^{-1}\left(\frac{H\lambda}{HD}\right) = \frac{\lambda_s - \lambda_y}{D_{\varphi_x} - D_{\varphi_s}} \tag{8}$$

where:

D_{φ_x} is the Meridional Part at φ_x :

$$D_{\varphi_x} = a \ln \operatorname{tg}\left(\frac{\pi}{4} + \frac{\varphi_x}{2}\right) \left(\frac{1 - e \sin \varphi_x}{1 + e \sin \varphi_x}\right)^{\frac{e}{2}} \tag{9}$$

D_{φ_s} is the Meridional Part at φ_s :

$$D_{\varphi_s} = a \ln \operatorname{tg}\left(\frac{\pi}{4} + \frac{\varphi_s}{2}\right) \left(\frac{1 - e \sin \varphi_s}{1 + e \sin \varphi_s}\right)^{\frac{e}{2}} \tag{10}$$

a is semi-major axis of Earth ellipsoid

e is eccentricity of Earth ellipsoid

$$M_{xy}F = SM_{xy} - SE = h_s - h_{xy} \tag{11}$$

where:

$$h_{xy} = \sin^{-1}[\sin \varphi_x \sin \delta + \cos \varphi_x \cos \delta \cos(t_G \pm \lambda_{yW}^{yE})] \tag{12}$$

The most probable ship’s position (M_{mn}) satisfies the

following condition:

$$(M_{mn}F)^2 = \min\{(M_{xy}F)^2\} \tag{13}$$

4. Celestial Ship Positioning Program Construction

4.1. Flow chart of the Program

With the above mathematical methodology, the flow chart of ship positioning by simultaneous observation of altitude and azimuth of sun is proposed in Figure 7.

4.2. Procedure of the Program

Step 1: Calculate the estimated error, determine the radius of the circle of the position probability according to the requirements of navigation accuracy (> 95%).

Step 2: Determine the searching domain on the basis of the estimated position to ensure the probability of containing ship's position meeting the requirements of the International Maritime Organization (> 95%).

Step 3: Establish the set of assumed ship's positions in the searching domain with coordinates and reasonable order.

Step 4: Calculate the errors of the assumed ship's positions in the searching domain. These errors are calculated based on the distance to true position.

Step 5: Determine the most probable ship's position by Least Squared Method.

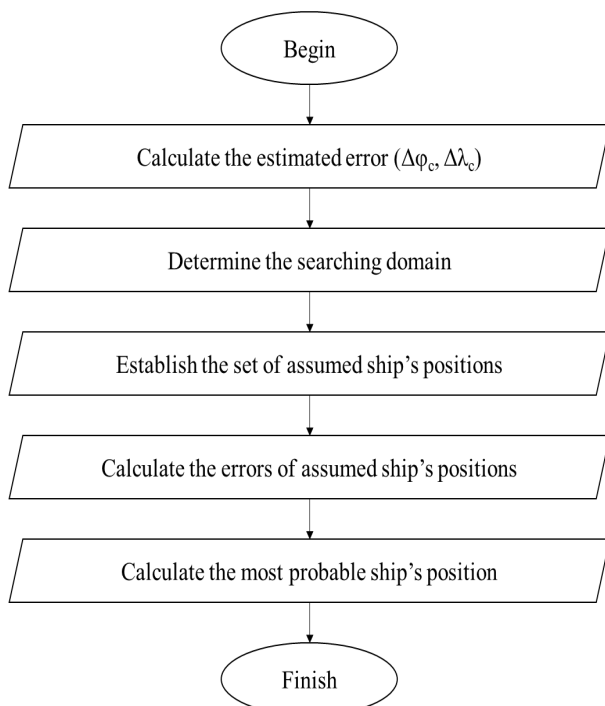


Figure 5: Flow chart of ship positioning by sun observation program

4.3. Ship Positioning by Sun Observation Program

Based on the proposed methodology, the automatic program to calculate the ship's position is constructed. The program is simply designed, suitable with the condition and ability of mariner. The main interface of the program is shown in Figure 8.

Compass Error (deg.)	0,1	Sun Navigation Fix Program			
Speed Log Error (%)	0,3				
GMT (hh,mm,ss)	090000	Degrees	Minutes	Second	Name
Nautical Almanac	GHA	132	44	30	E
	DEC	16	14	06	N
Dead Reckoning	Lat	20	12	45	N
	Long	107	50	39	E
Fixed Position	Lat	20	11	39	N
	Long	107	51	15	E

Figure 8: Ship Positioning by Sun Observation Program

4.4. Experiment on Board a Ship

The experiment to validate the program and compare with the traditional celestial method is carried out. On 18 September 2020, the MV. JEWEL OF SHINAS of New Ocean Ship Management PTE LTD is proceeding at 1800, the speed 12.1 kn. The sun crossed the observer's upper meridian at 11:54 according to the Nautical Almanac (2011).

- Before the moment the sun crossing the observer's upper meridian, at 10:30, the first observation of the sun was carried out. The altitude and azimuth of the sun were measured. The ship's position was fixed by the celestial simultaneous method. The estimated position was from GPS.

- Calculate the time period such that the azimuth variation is greater than 300, at 10:50, the second altitude and azimuth of the sun were measured. The ship's position was fixed by both the celestial simultaneous and non-simultaneous methods. The estimated position was determined by course and speed, not from GPS.

- After the moment the sun crossing the observer's upper meridian, at 13:30, the third altitude and azimuth of the sun were measured. The ship's position was fixed by the celestial simultaneous method. The estimated position was from GPS.

- Calculate the time period such that the azimuth

variation is greater than 300, at 13:50, the forth altitude and azimuth of the sun were measured. The ship’s position was fixed by both the celestial simultaneous and non-simultaneous methods. The estimated position was determined by course and speed, not from GPS.

The results are shown in Table 1-2. At each observation from 1 ÷ 4, the error of positions fixed by simultaneous observation of altitude and azimuth of sun method and positions from GPS are 2,6 – 2,5 – 2,7 – 2,9 nm. The accuracy of the position by celestial simultaneous method is high and stable, is not much influenced by the estimated error.

The error of positions fixed by traditional method of non-simultaneous observations of the sun and positions from GPS are 11,6 and 12,1 nm. The accuracy is low and is influenced by the estimated error.

From above-mentioned experiment, it can be shown that find that the new approach is possible to be applied on board a ship. In the future, this technique will be investigated with more experiments to verify its feasibility and effectiveness. Moreover, some external disturbances such as wind, current, and wave will be also considered to improve this celestial navigation technique.

Table 1: The result of ship positioning by the proposed simultaneous observation method and GPS position

GMT	GPS Position		Position by the proposed simultaneous observation method	
1030 18/9/2020	20°12'8 N	107°50'6 E	20°15'0 N	107°52'1 E
1050 18/9/2020	20°08'6 N	107°50'6 E	20°06'2 N	107°51'5 E
1330 18/9/2020	19°35'0 N	107°50'8 E	19°32'5 N	107°51'7 E
1350 18/9/2020	19°30'8 N	107°50'9 E	19°33'5 N	107°52'1 E

Table 2: The result of ship positioning by the non-simultaneous observation method and GPS position

GMT	GPS Position		Position by the non-simultaneous observation method	
1030 18/9/2020	20°12'8 N	107°50'6 E		
1050 18/9/2020	20°08'6 N	107°50'6 E	20°11'1 N	107°38'9 E
1330 18/9/2020	19°35'0 N	107°50'8 E		
1350 18/9/2020	19°30'8 N	107°50'9 E	19°39'7 N	107°59'5 E

5. Conclusions

This paper introduced the methodology of celestial ship position fixing method by non-simultaneous observation of the sun. Based on this traditional method, the authors proposed a new method of ship’s position determination by simultaneous observation of altitude and azimuth of the sun. The proposed method has shown some advantages such as simple, fast process and it can be carried out in the daytime. By the new methodology, the new algorithm and the program to automatically calculate the ship’s position are introduced. The experiment on board a ship of this program showed the effectiveness in comparison with other methods: high accuracy, stable and overcome the influence of the estimated error and external factors. With these advantages, this program can be easily applied on board. This method also ensures that the celestial ship positioning method is reliable backup method with high applicability.

6. Acknowledgements

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