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Original article

A Transportation Problem with Uncertain Costs and Random Supplies

Haiying GUO [1], *Xiaosheng WANG* [2] †, *Shaoling ZHOU* [1]

[1] School of Science, Hebei University of Engineering, China

[2]† School of Science, Hebei University of Engineering, China, xswang@hebeu.edu.cn, Corresponding Author

Abstract

Transportation problem is an optimization problem. In general, it was studied under random or uncertain condition. Considering the recent complexity, it is not enough to make should be a perfect transportation plan only based on. Usually, there is not only uncertainty but also randomness in many systems. In this paper, the aim is to investigate a transportation problem under uncertain and random environment. As a result, a conceptual uncertain random model is proposed for the problem, where the supplies are considered as random variables, and the costs and the demands are uncertain variables. By minimizing the expected value of uncertain objective function and taking confidence levels on constraints, transforming the model into a crisp mathematical form is the main conclusion. By minimizing the expected value of uncertain objective function and taking confidence levels on constraints, the above model can be turned to a mathematical form. Then transforming the model into a typical mathematical programming model is the main conclusion by using uncertainty theory and probability theory. At the end, a numerical example is given to show the feasibility of the model.

Keywords: Transportation problem, Uncertainty theory, Uncertain programming, Uncertain random variable

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I. Introduction

In recent years, with the development of economical globalization, goods transportation problems are focused on by more and more companies and enterprises, especially many multinational corporations. Transportation problem is an optimization problem which aims at the optimal distribution of the quantities from several sources to some destinations to minimize the total cost. In general, a traditional transportation model consists of an objective function and two kinds of constraints, namely source constraint and destination constraint. It was initiated by Hitchcock (1941) and later developed by Koopmans (1947). After four years, Dantzig (1951) proposed the simplex method and applied it to solve the transportation models. After that, transportation problems were studied by many researchers. Srinivasan and Thompson (1972) presented an operator theory of parametric programming for transportation problem. In 1991, Vignaux and Michalewicz (1991) gave a genetic algorithm for the linear transportation problem. In these classical models, the unit costs of transportation, the supplies and the demands were supposed to be crisp numbers.

Considering the various complexity in real world, some researchers were aware of a fact that it was usually inappropriate to regard the unit costs of transportation, the supplies and the demands as crisp numbers. They should be considered as variables. Thus, Williams (1963) assumed them to be random variables and built a random model for solving the kind of transportation problems. Since then, subsequent researchers began to study the stochastic transportation problem. In 1995, Wilson (1995) proposed a mean cost approximation method to solve the model. At the same year, Holmberg (1995) presented efficient decomposition and linearization methods for the stochastic transportation problem.

Along with the development of study, some researchers found that no investigated data are available to estimate a probability distribution in many cases. For instance, when the decision-makers meet new cooperators coming from unacquainted cities, it is impossible to obtain the statistical data of the parameters such as the unit costs of transportation in transportation problem. While a fundamental premise of applying probability theory is that the estimated probability distribution is close enough to the long-run cumulative frequency. In other words, probability theory has no effect in the case of shortage of sufficient observed data. Thus they have no choice but to invite some domain experts to evaluate the above parameters. Therefore, for dealing with human uncertainty, uncertainty theory was founded by Liu (2007) and refined by Liu (2010) based on normality, duality, subadditivity and product axioms. Nowadays, uncertainty theory has become a branch of axiomatic mathematics for modeling belief degrees. First, Liu (2007, 2009a) introduced the uncertain measure satisfying normality axiom, duality axiom, subadditivity axiom, and product axiom. Now uncertain measure has become a powerful tool to deal with belief degrees in uncertainty theory. Next, as a fundamental concept in uncertainty theory, uncertain variable was given by Liu (2007) to indicate the quantities with uncertainty. In fact, it is difficult to exactly describe an uncertain variable in many practical cases. Thus uncertainty distribution was presented by Liu (2007). Usually once uncertainty distribution is given, many properties of uncertain variable are easily obtained. Finally, the operational law was established by Liu (2010) to calculate the uncertainty distribution and inverse uncertainty distribution of strictly monotone function of independent uncertain variables. As an important contribution, Liu and Ha (2010) derived a useful formula for calculating the expected values of strictly monotone function of independent uncertain variables. Up to date, uncertainty theory has been a completely mathematical system.

Uncertain programming was founded by Liu (2009b). Since then, it was widely applied to deal with uncertain problems by many researchers. Sheng and Yao (2012a, 2012b) presented a transportation model with uncertain costs and demands and an uncertain programming model for fixed charge transportation problem in 2012. Meantime, Cui and Sheng (2012) proposed an uncertain model for solid transportation problem.

In practice, though the unit costs of transportation and the demands are uncertain, the supplier might usually predict the distribution of capacities of the supply according to statistical data of production capacity over the years. Thus for the supplier, capacities of the supply should be regarded as random variables rather than uncertain variables. For the above reasons, the main purpose of this paper is to study a transportation model with random supplies and uncertain unit costs and uncertain demands.

This paper consists of 6 sections. An outline is as follows. In Section 2, some basic concepts and results are presented. In Section 3, a conceptual transportation model with uncertain costs and random demands is constructed. By uncertainty theory and probability theory, the crisp equivalent form of the model is proposed in Section 4. After that, a numerical example is given in Section 5. The last section is a brief summary about the paper.

II. Preliminaries

In this section, we give some necessary concepts and results in uncertainty theory.

Let Γ be a nonempty set, and L be a σ-algebra over Γ. Each element $\lambda \in L$ is called an event. A number $M\{\Lambda\}$ indicates the level that Λ will occur. Uncertain measure M was introduced as a set function satisfying the following four axioms (Liu, 2007, 2009a):

Axiom 1. $M\{\Gamma\}=1$ for the universal set Γ.

Axiom 2. $M\{\Lambda\}+M\{\Lambda^c\}=1$ for any event Λ.

Axiom 3. For every countable sequence of events $\Lambda_1, \Lambda_2, \cdots$, we have

$$M\left\{\bigcup_{i=1}^{\infty} \Lambda_i\right\} \le \sum_{i=1}^{\infty} M\{\Lambda_i\}. \tag{1}$$

Note that the triplet (Γ, L, M) is called an uncertainty space.

Axiom 4. Let (Γ_k, L_k, M_k) be uncertainty spaces for $k = 1, 2, \cdots$. The product uncertain measure M is an uncertain measure satisfying

$$M\left\{\prod_{k=1}^{\infty} \Lambda_k\right\} = \bigwedge_{k=1}^{\infty} M\{\Lambda_k\} \tag{2}$$

where Λ_k are arbitrarily chosen events from L_k for $k = 1, 2, \cdots$, respectively.

Definition 1 (Liu, 2007) The uncertain variable ξ is a function from an uncertainty space (Γ, L, M) to the set of

real numbers such that $\xi \in B$ is an event for any Borel set B.

Definition 2 (Liu, 2007) Uncertainty distribution $\Phi : R \to [0,1]$ of an uncertain variable ξ is defined as $\Phi(x) = M\{\xi \le x\}$. (3)

Definition 3 An uncertain variable ξ is called normal if it has a normal uncertainty distribution

$$\Phi(x) = \left(1 + \exp\left(\frac{\pi(e-x)}{\sqrt{3}\sigma}\right)\right)^{-1}, x \in R \tag{4}$$

denoted by $N(e, \sigma)$ where e, σ are real numbers with $\sigma > 0$.

Definition 4 (Liu, 2010) Let ξ be an uncertain variable with regular uncertainty distribution $\Phi(x)$. Then the inverse function $\Phi^{-1}(\alpha)$ is called the inverse uncertainty distribution of ξ.

Clearly, the normal uncertain variable $N(e, \sigma)$ has a regular uncertainty distribution and its inverse uncertainty distribution is

$$\Phi^{-1}(\alpha) = \exp\left(e + \frac{\sigma\sqrt{3}}{\pi} In \frac{\alpha}{1-\alpha}\right). \tag{5}$$

We assume all uncertainty distributions in this paper are regular.

Theorem 1 (Liu, 2010) Let $\xi_1, \xi_2, \cdots, \xi_n$ be independent uncertain variables with regular uncertainty distributions $\Phi_1, \Phi_2, \cdots, \Phi_n$, respectively. If the function $f(x_1, x_2, \cdots, x_n)$ is strictly increasing with respect to $x_1, x_2, \cdots, x_m$ and strictly decreasing with respect to $x_{m+1}, x_{m+2}, \cdots, x_n$, then the uncertain variable $\xi = f(\xi_1, \xi_2, \cdots, \xi_n)$ has an inverse uncertainty distribution

$$\Psi^{-1}(\alpha) = f\left(\Phi_1^{-1}(\alpha), \cdots, \Phi_m^{-1}(\alpha), \Phi_{m+1}^{-1}(1-\alpha), \cdots, \Phi_n^{-1}(1-\alpha)\right). \tag{6}$$

In order to rank the uncertain variables, the expected value of the uncertain variable is introduced as follows.

Definition 5 (Liu, 2007) Let ξ be an uncertain variable. Then the expected value of ξ is defined by

$$E(\xi)=\int_0^{+\infty} M\{\xi \ge x\}\,dx-\int_{-\infty}^{0} M\{\xi \le x\}\,dx \tag{7}$$

provided that at least one of the two integrals is finite.

Easily, we can show that the normal uncertain variable $N(e,\sigma)$ has an expected value e, i.e., $\mathrm{E}(\xi) = e$.

Theorem 2 (Liu and Ha, 2010) Assume $\xi_1, \xi_2, \cdots, \xi_n$ are independent uncertain variables with regular uncertainty distributions $\Phi_1, \Phi_2, \cdots, \Phi_n$, respectively. If $f(x_1, x_2, \cdots, x_n)$ is strictly increasing with respect to $x_1, x_2, \cdots, x_m$ and strictly decreasing with respect to $x_{m+1}, x_{m+2}, \cdots, x_n$, then the uncertain variable $\xi = f(\xi_1, \xi_2, \cdots, \xi_n)$ has an expected value

$$E(\xi)=\int_0^1 f\left(\Phi_1^{-1}(\alpha),\cdots,\Phi_m^{-1}(\alpha),\Phi_{m+}^{-1}(1-\alpha),\cdots,\Phi_n^{-1}(1-\alpha)\right)d\alpha \tag{8}$$

Assume that x is a decision vector, and ξ is an uncertain vector. For given confidence levels $\alpha_1, \alpha_2, \cdots, \alpha_p$, Liu (2009b) proposed the following programming model,

$$\begin{cases} \min\limits_{x} \; E\left[(x,\xi)\right] \\ \textit{subjet to}: \\ \quad M\left\{g_j(x,\xi)\le 0\right\}\ge\alpha_j, \;\; j=1,2,\cdots,p. \end{cases} \tag{9}$$

Theorem 3 (Liu, 2009b) Assume the constraint function $g(x, \xi_1, \xi_2, \cdots, \xi_n)$ is strictly increasing with respect to $\xi_1, \xi_2, \cdots, \xi_k$ and strictly decreasing with respect to $\xi_{k+1}, \xi_{k+2}, \cdots, \xi_n$. If $\xi_1, \xi_2, \cdots, \xi_n$ are independent uncertain variables with regular uncertainty distributions $\Phi_1, \Phi_2, \cdots, \Phi_n$, respectively, then the chance constraint

$$M\{g(x, \xi_1, \xi_2, \cdots, \xi_n) \le 0\} \ge \alpha \tag{10}$$

holds if and only if

$$g\left(x, \Phi_1^{-1}(\alpha), \cdots, \Phi_k^{-1}(\alpha), \Phi_{k+1}^{-1}(1-\alpha), \cdots, \Phi_n^{-1}(1-\alpha)\right) \le 0. \tag{11}$$

III. Problem Description

Suppose that there are m sources, n destinations in a transportation problem. The goal of the problem is to make a transportation plan so that the total transportation cost is minimized. Let c_{ij} denote the costs of unit transportation amounts from sources i to destinations j and x_{ij} the capacities transported from sources i to destinations j, $i = 1, 2, \cdots, m$, $j = 1, 2, \cdots, n$, respectively. The capacities of sources i and the minimal demands of destinations j are denoted by a_i and b_j, $i = 1, 2, \cdots, m$, $j = 1, 2, \cdots, n$, respectively. Therefore, the transportation problem can be described as follows:

$$\begin{cases} \min \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \\ subjet\ to: \\ \quad \sum_{j=1}^{n} x_{ij} \le a_i, \ i=1, 2, \cdots, m \\ \quad \sum_{i=1}^{m} x_{ij} \ge b_j, j=1, 2, \cdots, n \\ \quad x_{ij} \ge 0, i=1, 2, \cdots, m, \ j=1, 2, \cdots, n. \end{cases} \tag{12}$$

In this model, the first constraint suggests that the total capacities transported from sources i are no more than the supply capacities a_i of sources i and the other constraint implies that the total amounts transported to destinations j should satisfy the demands of j, $i = 1, 2, \cdots, m$, $j = 1, 2, \cdots, n$, respectively. In the real world, as the transportation programming is needed to make in advance, so the decision-makers might meet various uncertainty, including for instance weather factor and traffic factor. As a result, the parameters in the above model are unknown and indeterminate. Thus for the suppliers, the decision-makers could make a prediction to the capacities of supplying according to the capacity of production in the past years. That is to say, the capacities of supplying should be regarded as random variables. But facing to the new demanders, the decision-makers could make a prediction to the capacities of demanding and the unit costs of transportation only depending on the experts' data. Hence based on the above reasons, we may assume that c_{ij} and b_j are independent uncertain variables, and a_i are random variables, $i = 1, 2, \cdots, m$, $j = 1, 2, \cdots, n$.

IV. Mathematical Model

Based on the above assumption, it is clear that the model (12) is only a conceptual model as there is not an ordership with respect to the uncertain variable and random variable. Hence in the model (12), we take the expected value on the objective function and confidence levels on the constraint conditions. Then we obtain its equivalent form as follows,

$$\begin{cases} \min \mathrm{E}\left[\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}\right] \\ \textit{subjet to}: \\ \quad P\left[\sum_{j=1}^{n} x_{ij} \le \mathrm{a}_i\right] \ge \alpha_i,\ \ i=1,2,\cdots,m \\ \quad M\left\{\sum_{i=1}^{m} x_{ij} \ge b_j\right\} \ge \beta_j,\ \ j=1,2,\cdots,n \\ \quad x_{ij} \ge 0,\ i=1,2,\cdots,m,\ j=1,2,\cdots,n. \end{cases} \tag{13}$$

where α_i and β_j are the predetermined probability and uncertainty confidence levels for $i = 1, 2, \cdots, m$, $j = 1, 2, \cdots, n$, respectively. Applying probability theory and uncertainty theory, the model (18) is converted into an equivalent crisp mathematical model.

Theorem 4 Assume b_j and c_{ij} are independent uncertain variables with regular uncertainty distributions Φ_{b_j} and $\Phi_{c_{ij}}$, $i = 1, 2, \cdots, m$, $j = 1, 2, \cdots, n$, respectively. And assume a_i are random variables with regular probability distributions Ψ_{a_i}, $i = 1, 2, \cdots, m$. Then the model (13) has an equivalent crisp mathematical model

$$\begin{cases} \min \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij}\int_0^1 \Phi_{c_{ij}}^{-1}(\alpha)d\alpha \\ \textit{subjet to}: \\ \quad \sum_{j=1}^{n} x_{ij} \le \Psi_{\mathrm{a}_i}^{-1}(1-\alpha_i),\ \ i=1,2,\cdots,m \\ \quad \sum_{i=1}^{m} x_{ij} \ge \Phi_{b_j}^{-1}(\beta_j),\ \ j=1,2,\cdots,n \\ \quad x_{ij} \ge 0,\ i=1,2,\cdots,m,\ j=1,2,\cdots,n. \end{cases} \tag{14}$$

Proof: By Theorems 2, we have

$$\mathrm{E}\left[\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}\right] = \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij}E[c_{ij}] = \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij}\int_0^1 \Phi_{c_{ij}}^{-1}(\alpha)d\alpha.$$

We rewrite the inequality $M\left\{\sum_{i=1}^{m} x_{ij} \ge \mathrm{b_j}\right\} \ge \beta_j$ as follows,

$$M\left\{\mathrm{b_j} - \sum_{i=1}^{m} x_{ij} \le 0\right\} \ge \beta_j.$$

Using Theorem 3, the constraint condition $M\left\{\sum_{i=1}^{m} x_{ij} \geq \mathrm{b}_{\mathrm{j}}\right\} \geq \beta_j$ is equivalent to the following inequality

$$\Phi_{b_j}^{-1}(\beta_j) - \sum_{i=1}^{m} x_{ij} \leq 0 \text{ for } j = 1, 2, \cdots, n.$$

Deforming another constraint condition for each i, we have

$P\left[\mathrm{a}_i \geq \sum_{j=1}^{n} x_{ij}\right] \geq \alpha_i,\ i=1, 2, \cdots, m$. It follows that $1-P\left[\mathrm{a}_i \leq \sum_{j=1}^{n} x_{ij}\right] \geq \alpha_i$. It is equivalent to the inequality $\sum_{j=1}^{n} x_{ij} - \Psi_{\mathrm{a}_i}^{-1}(1-\alpha_i) \leq 0$, for $i = 1, 2, \cdots, m$.

Therefore, the theorem is verified.

V. Numerical Experiment

In this section, we shall provide a practical transportation problem to show the application of the model (19).

As we know, coal is an important material and is used widely in many fields. But the coal mines only gather in several provinces and the plants needing the coal are scattered everywhere. Hence it is inevitable to consider the transportation problem of coal. Assume that there are four coal mines and six new electric power plants. Now the decision-makers of the suppliers need to make a transportation plan for the next quarter in advance. According to the statistical production data, the capacities of supply a_i from mines i are regarded as random variables and obey the normal probability distributions $N(\mu_i, \sigma_i)$, $i = 1, 2, 3, 4$. Depending on the experts' data, the demanding capacities b_j of plants j and the unit costs c_{ij} of transporting from mines i to plants j are considered as uncertain variables and obey the normal uncertainty distributions $N\left(e'_j, \sigma'_j\right)$ and $N\left(\mathrm{e}'_{ij}, \sigma'_{ij}\right)$, $i = 1, 2, 3, 4$, $j = 1, 2, \cdots, 6$, respectively. The following three tables provide the values of the parameters in all distributions.

Table 1: Parameters of probability normal distribution $N(\mu_i, \sigma_i)$ of supplies

i	1	2	3	4
(μ_i, σ_i)	$(32, 1{,}5)$	$(38, 1{,}5)$	$(30, 2)$	$(29, 2)$

Table 2: Parameters of uncertainty normal distribution $N\left(e'_j, \sigma'_j\right)$ of demands

j	1	2	3	4	5	6
$\left(e'_j, \sigma'_j\right)$	$(10, 1{,}5)$	$(15, 1)$	$(20, 1)$	$(12, 1)$	$(14, 2)$	$(10, 1)$

Table 3: Parameters of uncertainty normal distribution $N\left(e'_{ij}, \sigma'_{ij}\right)$ of unit costs

$\left(e'_{ij}, \sigma'_{ij}\right)$	1	2	3	4	5	6
1	$(18, 2)$	$(17, 1{,}5)$	$(16, 2)$	$(17, 1{,}5)$	$(18, 2)$	$(8, 1{,}5)$
2	$(8, 1)$	$(9, 1{,}5)$	$(5, 2)$	$(18, 2)$	$(8, 1{,}5)$	$(18, 1{,}5)$
3	$(8, 1{,}5)$	$(16, 1{,}5)$	$(6, 1{,}5)$	$(10, 1{,}5)$	$(18, 1{,}5)$	$(20, 1{,}5)$
4	$(19, 1{,}5)$	$(12, 1{,}5)$	$(18, 1{,}5)$	$(10, 1{,}5)$	$(12, 1{,}5)$	$(20, 2)$

For $m=4, n=6$, applying Theorem 4, the corresponding equivalent model is

$$\begin{cases} \min \sum_{i=1}^{4} \sum_{j=1}^{6} x_{ij} e_{ij} \\ \textit{subjet to}: \\ \sum_{j=1}^{6} x_{ij} \le \mu_i + \sigma_i K_{1-\alpha_i}, \quad i=1,2,3,4 \\ \sum_{i=1}^{4} x_{ij} \ge e'_j + \frac{\sqrt{3}\sigma'_j}{\pi} In \frac{\beta_j}{1-\beta_j}, \quad j=1,2,\cdots,6 \\ x_{ij} \ge 0, i=1,2,3,4, j=1,2,\cdots,6. \end{cases} \tag{15}$$

where $K_{1-\alpha_i}$ are the $1-\alpha_i$ quantiles of the standard normal probability distribution, $i = 1, 2, 3, 4$, $j = 1, 2, \cdots, 6$.

Now the above model is a classic linear programming problem and its feasible region is a convex polyhedron.

Assume that the confidence levels are α_i=0.9 and β_j=0.9, $i = 1, 2, 3, 4$, j=1,2,$\cdots$,6. By means of MATLAB software, the optimal transportation plan is x_{16}=11.2114, x_{22}=14.0687, x_{23}=5.5885, x_{25}=16.4228, x_{31}=11.8171, x_{33}=15.6229, x_{42}=2.1427, x_{44}=13.2114, and the other decision variables are all zero.

VI. Conclusions

In this paper, a new transportation model was constructed based on uncertainty theory and probability theory. By taking expected value on the objective function and confidence levels on the constraint functions, it was converted into a crisp mathematical model. At last, a numerical experiment was given and its optimal solution was also found by the simplex method to show the feasibility of the model for the transportation problem.

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