Available online at www.sciencedirect.com

ScienceDirect

International Journal of e-Navigation and Maritime Economy 3 (2015) 181 – 193

International Journal of e-Navigation and Maritime Economy

www.elsevier.com/locate/enavi

Original article

An uncertain model for RCPSP with solution robustness focusing on logistics project schedule

Hua KE [1], *Lei WANG* [2*], *Hu HUANG* [3]

[1] School of Economics and Management, Tongji University, China, hke@tongji.edu.cn

[2*] School of Economics and Management, Tongji University, China, 1333205mlwlyy@tongji.edu.cn, Corresponding author

[3] School of Economics and Management, Tongji University, China, 13huanghu@tongji.edu.cn

Abstract

Logistics project scheduling problem in indeterminate environment is gaining more and more attention in recent years. One effective way to cope with indeterminacy is to develop robust baseline schedule. There exist many related researches on building robust schedule in stochastic environment, where historical data is sufficient to learn probability distributions. However, when historical data is not enough, precise estimation on variables may be impossible. This kind of indeterminate environment can be described by uncertainty according to uncertainty theory. Related researches in uncertain environment are sparse. In this paper, our aim is to solve robust project scheduling in uncertain environment. The specific problem is to develop robust schedule with uncertain activity durations for logistics project. To solve the problem, an uncertain model is built and an intelligent algorithm based on simulated annealing is designed. Moreover, we consider a logistics project as a numerical example and illustrate the effectiveness of the proposed model and algorithm.

Keywords: Logistics project scheduling; Uncertainty theory; Robust scheduling; Solution robustness; Simulated annealing

I. Introduction

Project scheduling is a core part of project management, which aims to control project makespan and cost as planed and to ensure project success. Generally, project scheduling is to assign activity starting time (and finishing time) for each activity subject to precedence relation constraint and resource constraint. The problem can be extended to many subproblems, such as resource constrained project scheduling problem (RCPSP), time-cost trade-off problem (TCTP), and resource leveling problem (RLP). As a standard type of project scheduling, RCPSP has been widely studied since Pritsker et al. (1969) presented a mathematical model for RCPSP. Typically, RCPSP aims at minimizing makespan in deterministic environment, where all information is known in advance and supposed to be deterministic. We can get a baseline schedule, a list of activity starting times, with minimal makespan by solving this problem. The schedule has many important functions during project execution. It is the base of enterprise’s internal activities, for example, allocating resources for each activity (Mehta and Uzsoy, 1998). Furthermore, it is the base of enterprise’s external activities according to Wu et al. (1993), e.g., purchasing and signing contracts with subcontractors. However, during execution, the environment is full of indeterminacies, including accident, resource breakdown, unreliable deliveries, etc. As a result, the validity of such baseline schedule may be influenced seriously, which means the realized schedule is very different from the planed one. One feasible way to cope with indeterminacy is to consider related information when we develop project schedule and to minimize the difference between the realized schedule and the planed one.

As proposed by Herroelen and Leus (2005), scheduling under indeterminate environment can be classified into five basic approaches: reactive scheduling, stochastic project scheduling, fuzzy project scheduling, robust (proactive) scheduling and sensitivity analysis. Robust scheduling is different from the other approaches in whether a baseline schedule is developed. Specifically, robust scheduling contributes to a robust schedule, which is designed to be protected from disruptions as much as possible. Considering the important role played by baseline schedule, robust scheduling is chosen to be studied in this paper.

In Van de Vonder et al. (2005), two kinds of robustness were distinguished, quality robustness and solution robustness. Solution robustness means the insensitivity of scheduled activity starting times to unexpected disruptions. It is measured by the deviations between the planed activity starting times and the realized ones. In real project, it is hard to say that a baseline schedule can guide project execution if the deviation is large, which means the baseline schedule makes little sense. Therefore, solution robustness of schedule should be paid enough attention.

Fortunately, many researches exist in the field of project scheduling with solution robustness. The realization of robustness mainly relies on redundance-based technique, namely time buffer or resource buffer. These studies may be summarized as two main approaches. The first one focuses on procedures to build robust schedule. Some alternative procedures may contain adapted float factor (ADFF) heuristic (Herroelen and Leus, 2004), resource flow-dependent float factor (RFDFF) heuristic (Van de Vonder et al., 2006), virtual activity duration extension (VADE) algorithm (Van de Vonder et al., 2008) and starting time criticality (STC) algorithm (Van de

Vonder et al., 2008). These procedures devote to constructing an effective mechanism of assigning time buffers for each activities. Generally, one procedure consists of two parts: Firstly, the corresponding RCPSP is solved. Secondly, time buffers are added into the schedule achieved through the first part. The second approach features the existence of mathematical model with an objective to be optimized, which represents robustness and is called robustness measure (RM). Relative researches are committed to find effective RM. For these researches, readers may refer to Lambrechts et al. (2008a, b), Chtourou and Haouari (2008), and Khemakhem and Chtourou (2013).

Robust project scheduling has been studied from different viewpoints. However, these researches were mainly made in stochastic environment with a latent assumption that historical data is enough and precise estimation of variables' distributions is available. The inherent assumption may not hold when historical data is laking. Actually, in project, we are unable to get enough data if some activities are seldom or never performed considering the uniqueness of each project. We need to describe variables by new ways instead of probability distribution. One optional method is to utilize uncertainty theory, initiated by Liu (2007) and refined by Liu (2010). For the lack of historical data, some elements, such as activity durations, are estimated according to belief degree provided by experts. As far as we know, the new theory has been successfully applied to the following fields: option pricing problem (Chen, 2011), stock problem (Peng and Yao, 2011; Bhattacharyya et al, 2013), facility location problem (Gao, 2012), differential equation (Yao, 2013), differential games (Yang and Gao, 2013), assignment problem (Zhang and Peng, 2013), inventory problem (Qin and Kar, 2013), supply chain pricing problem (Huang and Ke, 2014), etc. Specially, applications of uncertainty theory can be found in project scheduling. For detail, Zhang and Chen (2012) built an uncertain model to minimize expected project makespan. Ke et al. (2015) considered environment with uncertainty and randomness simultaneously and proposed an uncertain random model for project scheduling. Some other relevant researches include Ji and Yao (2014), and Ke (2014a, b). However, no research with uncertainty theory pays attention to robustness of schedule. To fill this gap, this paper studies robust project scheduling with uncertain activity durations. In detail, logistics project is considered and one uncertain model for robust project scheduling is proposed. And an intelligent algorithm based on simulated annealing (SA) is designed to solve the proposed model.

This paper is organized as follows: In the following section, some basic concepts in uncertainty theory are introduced. In Section III, we briefly describe uncertain robust project scheduling with some corresponding assumptions. Section IV proposes an uncertain model. Then we attempt to transform the model into a crisp programming model. Furthermore, an intelligent algorithm, with uncertain simulation embedded, based on SA is designed for the proposed model in Section V. Section VI gives a numerical experiment to implement our algorithm. Some results are presented. Finally, some conclusions are drawn in Section VII.

II. Preliminary

Uncertainty theory was initiated by Liu (2007) to describe uncertain phenomena. Given that Γ is a nonempty set, $\mathcal{L}$ is a σ-algebra over Γ, and each element Λ in $\mathcal{L}$ is called an event. Uncertain measure is defined as a function from $\mathcal{L}$ to [0,1]. In detail, Liu (2007) introduced the concept of uncertain measure as follows:

Definition 1 *Liu (2007) The set function $\mathcal{M}$ is called an uncertain measure if it satisfies:*

(i) $\mathcal{M}\{\Gamma\} = 1$ *for the universal set* Γ.

(ii) $\mathcal{M}\{\Lambda\} + \mathcal{M}\{\Lambda^c\} = 1$ *for any event* Λ.

(iii) For every countable sequence of events $\Lambda_1, \Lambda_2, \cdots$, *we have*

$$\mathcal{M}\left\{\bigcup_{i=1}^{\infty} \Lambda_i\right\} \leq \sum_{i=1}^{\infty} \mathcal{M}\{\Lambda_i\}.$$

Besides, the product uncertain measure on the product σ-algebra $\mathcal{L}$ was defined by Liu (2009) as follows:

(4) Let $(\Gamma_k, \mathcal{L}_k, \mathcal{M}_k)$ *be uncertainty spaces for* $k = 1,2,\cdots$ *The product uncertain measure* $\mathcal{M}$ *is an uncertain measure satisfying*

$$\mathcal{M}\left\{\prod_{i=1}^{\infty} \Lambda_k\right\} \leq \wedge_{k=1}^{\infty} \mathcal{M}\{\Lambda_k\}$$

where Λ_k *are arbitrarily chosen events from* $\mathcal{L}_k$ *for* $k = 1,2,\cdots$, *respectively*.

With the concept of uncertain measure, uncertain variable can be defined as follows:

Definition 2. *Liu (2007) An uncertain variable is a measurable function* ξ *from an uncertainty space* $(\Gamma, \mathcal{L}, \mathcal{M})$ *to the set of real numbers, i.e., for any Borel set B of real numbers, the set*

$$\{\xi \in B\} = \{\gamma \in \Gamma | \xi(\gamma) \in B\}$$

is an event.

To describe uncertain variable in detail, the concept of uncertainty distribution is given.

Definition 3. *Liu (2007) The uncertainty distribution* Φ *of an uncertain variable* ξ *is defined by*

$$\Phi(x) = \mathcal{M}\{\xi \leq x\}.$$

What’s more, Liu (2010) defined Φ^{-1} as the inverse uncertainty distribution of uncertain variable. The expected value of uncertain variable is defined as:

Definition 4. *Liu (2007) Let* ξ *be an uncertain variable. The expected value of* ξ *is defined by*

$$E[\xi] = \int_0^{+\infty} \mathcal{M}\{\xi \geq r\} dr - \int_{-\infty}^{0} \mathcal{M}\{\xi \leq r\} dr$$

provided that at least one of the above two integrals is finite.

Actually, the expected value can be easily computed if the corresponding variable has inverse distribution according to the following theorem.

Theorem 1 *Liu (2010) Let* ξ *be an uncertain variable with regular uncertainty distribution* Φ. *If the expected value exists, then*

$$E[\xi] = \int_0^1 \Phi^{-1}(\alpha) d\alpha.$$

III. Problem description

Robust project scheduling aims at generating a project schedule with solution robustness, the ability to cope with unexpected disruptions, e.g., unexpected longer activity durations. In logistic project, robustness, or stability, is one important measure of schedule effectiveness. Generally, activity-on-the-node (AON) network can be employed to explain logic relations between activities in a project. Consider $G = (N, A)$, shown as Fig. 1, as an example, where the node set $N = \{1,2,\cdots,n\}$ represents activities, and the arc set A represents finish-start, zero-lag precedence relations between activities. Activities 1 and n mark project start and end, respectively, both of which are dummy activities and consume no resources and time. Each of the other activities requires some amount of resources and time.

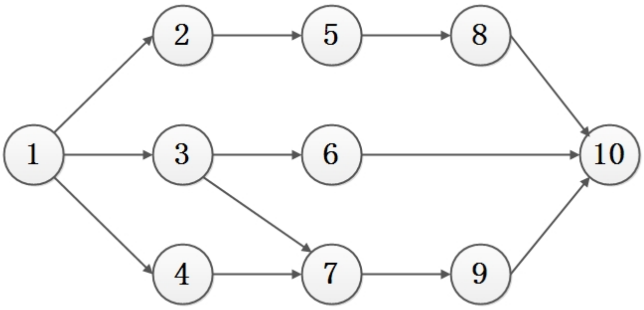

Figure 1: Project network

This paper focuses on robust project scheduling with uncertain activity durations, represented by an uncertain vector $\boldsymbol{d} = \{\widetilde{d_1}, \widetilde{d_2}, \cdots, \widetilde{d_{10}}\}$. An illustrative model is as follows:

$$Min \quad \sum_{i\in N} w_i |E[\mathbf{s_i}] - s_i| \tag{1}$$

$s.t.$

$$s_1 = 0 \tag{2}$$

$$s_i \geq s_j + inf(\widetilde{d_j}), \quad \forall(j,i) \in A \tag{4}$$

$$\sum_{i\in S_t} r_{ik} \leq a_k, \quad \forall t, \forall k \tag{3}$$

$$s_n \leq \delta \tag{5}$$

Objective function (1) denotes the weighted sum of deviations between realized activity starting times and planed ones. For detail, w_i denotes the marginal instability (deviation) cost of activity i, $\mathbf{s_i}$ is the realized starting time of activity i, and s_i is the planed starting time of activity i. Constraint (2) sets s_1 as 0, meaning the whole project starts at time 0. Constraint (3) shows that activity i can not start before the earliest finishing time of activity j if there exists a precedence relation between activities j and i. $inf(\widetilde{d_j})$ is lower bound of activity j's duration. This constraint is suitable for situation where floors of durations are determined. In constraint (4), a deadline δ is added. It means project end s_n can not exceed the deadline. This constraint is used to prevent the appearance of a robust schedule with an unconstrained long makespan. Formula (5) enforces renewable resource constraint, where S_t represents the set of underway activities at time t, r_{ik} is the demand of activity i for resource type k, and a_k is the given available amount of the kth resource. That is to say, the resource requirement of activities in process at any time is no more than the total available amount for any resource type.

Some correlative statements and assumptions include: (a) Uncertainty to be addressed in this paper is linear uncertain activity durations, denoted by linear uncertain variables $\widetilde{d_1}, \widetilde{d_2}, \cdots, \widetilde{d_n}$ with inverse uncertain distributions Φ_1^{-1}, Φ_2^{-1}, $\cdots$, Φ_n^{-1}; (b) Solution robustness is realized with the help of redundancy-based technique, more explicitly inserting buffer times to minimal activity durations; (c) Each activity is assumed to be executed in one mode and preemption is not permitted; (d) In objective function, the marginal instability cost w_i is a constant number; (e) In resource constraint, only one resource type is considered.

For the generation of realized activity starting times $(\mathbf{s_1}, \mathbf{s_2}, \cdots, \mathbf{s_n})$, railway scheduling rule is adopted, where activities are not allowed to start earlier than the planed starting times. Compared to railway scheduling, roadrunner scheduling starts feasible activities as soon as possible. Generally, roadrunner scheduling rule contributes to ending project before deadline (quality robustness) while pays little attention to solution robustness. A comparison between the two rules was made by Tian and Demeulemeester (2014). They found that railway scheduling is better than roadrunner scheduling in improving both stability (solution robustness) and expected project makespan (quality robustness) in one realistic situation. We choose railway scheduling considering its function in increasing solution robustness of a schedule.

IV. Uncertain model for robust project scheduling

Robust project scheduling aims to maximize schedule robustness. The conceptual model presented in Section III shows that maximizing solution robustness is converted to minimizing the weighted sum of deviations between realized activity starting times and planed ones. However, realized activity starting times are not available because they are subject to uncertain activity durations. Expected activity starting times are employed to get deviations. As mentioned in Section III, railway schedule is adopted in the process of generating realized activity starting times, suggesting that realized activity starting times are all not less than the corresponding planed ones. Thus we can remove absolute value sign in the objective. Furthermore, to violate the deadline constraint, we add a penalty function in objective function. Finally, the objective can be rewritten as:

$$Min \quad E[\sum_{i \in N} w_i(\mathbf{s_i} - s_i) + p(0, s_n - \delta)^+]. \tag{5}$$

Obviously, objective function (6) is an expected value, where p is an extra penalty which appears when project end time in schedule exceeds the deadline. For project manager indifferent to risk, it is reasonable to minimize this expected value. For simplification, we define c_s as

$$c_s = \sum_{i \in N} w_i(\mathbf{s_i} - s_i) + p(0, s_n - \delta)^+.$$

Next, we try to transform the proposed model to a crisp programming model. If resource constraint is absent, realized starting time of activity i can be computed by

$$\mathbf{s_i}(s, \mathbf{d}) = s_i \vee \max_{(j,i) \in A}(\mathbf{s_j}(s, \mathbf{d}) + \widetilde{d_j})$$

where $s = (s_1, s_2, \cdots, s_n)$ is a planed schedule.

However, this formula is invalid when resource constraint is considered. Some activity may be feasible in term of precedence relation when all of its predecessors have been finished while infeasible for the lack of available resource. In other words, one activity has predecessors in both precedence relation logic and resource logic. To get schedule precedence feasible and resource feasible, resource flow network was proposed in Artigues and Roubellat (2000), where extra relations are added into the original AON network if there exists a resource flow between two activities without precedence relations. We define the extended precedence relation set as A^*. Then realized starting time of activity i , a function of schedule and uncertain activity durations, can be calculated by

$$\mathbf{s_i}(s, \mathbf{d}) = s_i \vee \max_{(j,i) \in A^*}(\mathbf{s_j}(s, \mathbf{d}) + \widetilde{d_j}).$$

Provided that $\mathbf{s_i}(s, \mathbf{d})$ has an inverse uncertainty distribution $\Psi_i^{-1}(s, \alpha)$, $\alpha \in (0,1]$, we can get

$$\Psi_i^{-1}(s, \alpha) = s_i \vee \max_{(j,i) \in A^*}(\Psi_j^{-1}(s, \alpha) + \Phi_j^{-1}(\alpha)).$$

The sum of instability costs c_s can be simply calculated by

$$c_s(s, \mathbf{d}) = \sum_{i \in N} w_i(\mathbf{s_i}(s, \mathbf{d}) - s_i) + p(0, s_n - \delta)^+$$

with an inverse uncertainty distribution

$$\gamma^{-1}(s, \alpha) = \sum_{i \in N} w_i(\Psi_i^{-1}(s, \alpha) - s_i) + p(0, s_n - \delta)^+.$$

Accordingly, the uncertain model for robust project scheduling can be rewritten as

$$Min \quad \int_0^1 \gamma^{-1}(s, \alpha) d\alpha + p(0, s_n - \delta)^+ \tag{6}$$

$s.t.$

$$s_1 = 0$$
$$s_i \geq s_j + inf(\widetilde{d_j}), \quad \forall (j,i) \in A$$
$$\sum_{i \in S_t} r_{ik} \leq a_k, \quad \forall t, \forall k.$$

V. Intelligent algorithm

Blazewicz et al. (1983) proved the deterministic RCPSP is strongly NP-hard, letting alone the extension of RCPSP involving uncertain activity durations. Therefore, intelligent algorithm based on SA are designed to solve the proposed model. To get across our algorithm, solution representation, uncertain simulation based on 99-method and algorithm outline are introduced in detail.

5.1 Solution representation

There exist many feasible solution representations in project scheduling, as discussed in Kolisch and Hartmann (1999). We adopt one representation consisting of an ordered list of activities AL and a corresponding buffer time list BL, which is similar with that of Lambrechts et al. (2008b). The difference is that we add buffer times behind activities instead of before activities like Lambrechts et al. (2008b). Thus the assigned duration for each activity includes the duration lower bound and its buffer time. In general, there are two reasons for the selection: Firstly, this representation can be exactly and easily decoded into a feasible schedule s by Algorithm 1. Secondly, adjusting AL and BL, operated in mutation and crossover, can availably find the neighbourhood of the solution.

Algorithm 1: Decoding process

AL:= activity list: $[L_1, L_2, L_3, \cdots, L_n]$
BL:= buffer time list: $[B_1, B_2, B_3, \cdots, B_n]$
$PRED(i)$:= the direct predecessors of activity i
$\tilde{d_i}$:= the uncertain duration of activity i
for $i = 1{:}n$
    if isempty$(PRED(AL(i))) = 1$
        $s_{AL(i)} = 0$
    else
        $s_{AL(i)} = \max_{j \in PRED(AL(i))}(f_j)$
    end
    while $\exists k, t{:} \sum_{i \in S_t} r_{ik} > a_k$ do
        $s_{AL(i)} = s_{AL(i)} + 1$
    end
    $f_{AL(i)} = s_{AL(i)} + inf\tilde{d}_{AL(i)}) + B_{AL(i)}$
end
s_n=$max(s_n, \delta)$

5.2 Uncertain simulations

Objective function in this paper is transformed to function (7). However, we cannot get specific function form because resource flow in each schedule is different. As a result, extended precedence relation set A^* changes with specific schedule and $\mathbf{s_i}(s, \mathbf{d})$ is not of one specific form. Fortunately, uncertain simulation based on 99-method has been developed by Liu (2010). This method has been successfully used in project scheduling by Zhang and Chen (2012).

Assume $\tilde{d_i}$, the uncertain duration of activity i, is represented by a 99-table

0.01	0.02	0.03	⋯	0.99
d_i^1	d_i^2	d_i^3	⋯	d_i^{99}

Then we have 99-table of $\mathbf{s_i}(s, \boldsymbol{d})$

0.01	0.02	0.03	⋯	0.99
$\boldsymbol{s_i}^1$	$\boldsymbol{s_i}^2$	$\boldsymbol{s_i}^3$	⋯	$\boldsymbol{s_i}^{99}$

Accordingly, the sum of instability costs $\mathbf{c_s}(s, \boldsymbol{d})$ can be described by a 99-table as

0.01	0.02	⋯	0.99
$\sum_{i\in N} w_i(\boldsymbol{s_i}^1 - s_i) + p(0, s_n - \delta)^+$	$\sum_{i\in N} w_i(\boldsymbol{s_i}^2 - s_i) + p(0, s_n - \delta)^+$	⋯	$\sum_{i\in N} w_i(\boldsymbol{s_i}^{99} - s_i) + p(0, s_n - \delta)^+$

The uncertain simulation based on 99-method for $E[c_s]$ is presented in Algorithm 2.

Algorithm 2: Uncertain simulation for $E[c_s]$

Step 1: Get s_i by decoding process, where $i = 1,2,\cdots,n$.

Step 2: Generate $d_1^m, d_2^m, \cdots, d_n^m$ according to the distribution of uncertain variables $\widetilde{d_1}$, $\widetilde{d_2}$, ⋯, $\widetilde{d_n}$, and denote $\boldsymbol{d^m}$=$(d_1^m, d_2^m, \cdots, d_n^m)$, m=1,2,⋯,99, respectively.

Step 3: Get $\boldsymbol{s_i}^m$ according to s_i and $\boldsymbol{d^m}$, m=1,2,⋯,99.

Step 4: Compute c_s^m=$\sum_{i\in N} w_i(\boldsymbol{s_i}^m - s_i) + p(0, s_n - \delta)^+$, m=1,2,⋯,99.

Step 5: $E[c_s]$=$\sum_{m=1}^{99} c_s^m/99$.

5.3 Algorithm based on SA

In this subsection, intelligent algorithm is proposed by embedding uncertain simulation into SA. This algorithm is designed to update a feasible solution to the best solution. Fist, an initial activity list AL is gained by solving the condescending project scheduling problem, and buffer time list BL is given arbitrarily. Second, buffer time list BL is updated in the process of algorithm based on SA. Some corresponding parameters are as follows:

T_0: One crucial control parameter in SA, expressing initial temperature.

L: Another crucial control parameter in SA, denoting reputation number at each temperature.

T_f: Freezing time, which is related with stopping criterion.

C: A coefficient used in temperature reducing function.

AL: Current activity list.

BL: Current buffer time list.

BL^*: Buffer time list in historical best solution.

The outline of the intelligent algorithm based on SA is presented in Algorithm 3.

Algorithm 3: The outline of the intelligent algorithm based on SA

Set $T = T_0$, $AL = AL_0$, $BL = BL_0$, $BL^* = BL_0$.

According to AL and BL, calculate $E[c_s]$. Set $E = E[c_s]$, $E^* = E[c_s]$.

```
while T ≥ T_f do
    for m = 1:L
        Randomly select an activity a and get BL' by altering the ath element of BL
        According to AL and BL', calculate E' = E[c_s]
        Δ=E' − E
        if Δ ≤ 0
            BL = BL', E = E'
        else
            Generate a random number r from (0,1)
            if r ≤ e^(−Δ/T)
                BL = BL', E = E'
            end
        end
        if E ≤ E*
            BL* = BL, E* = E
        end
    end
    T = C * T
end
```

VI. Numerical experiment

In this section, a logistics project shown in Fig. 1 is chosen to execute the intelligent algorithm. Table 1 gives relative basic data such as activity duration, resource requirement and marginal deviation cost of each activity. Specifically, activity duration is represented by linear uncertain variable $\mathcal{L}(a, b)$ with $a < b$. Project deadline δ is set as 185. And penalty p in objective is assumed to be 1000.

Table 1: Basic data of activities

Node	Activity duration	Resource requirement	Marginal deviation cost	Node	Activity duration	Resource requirement	Marginal deviation cost
1	0	0	0	6	$\mathcal{L}(50,70)$	2	1
2	$\mathcal{L}(15,30)$	5	15	7	$\mathcal{L}(30,50)$	3	1
3	$\mathcal{L}(30,50)$	4	12	8	$\mathcal{L}(60,90)$	3	9

4	$\mathcal{L}(60,90)$	3	10	9	$\mathcal{L}(10,30)$	4	5
5	$\mathcal{L}(25,40)$	4	11	10	0	0	38

The intelligent algorithm based on SA is operated. Control parameters are set as: $T_0 = 2000$, $L = 8$, $T_f = 0.005$ and $C = 0.95$. The input activity list $AL = (1,2,3,4,6,7,8,5,9,10)$ is the best solution in a genetic algorithm for robust project scheduling. And the input buffer time list BL is $(0,0,0,0,0,0,0,0,0,0)$. As showed in Fig. 2, after improving for about 80 times, we get a quasi-optimal solution. For detail, we list activity list AL, buffer time list BL, the correlative schedule s and the quasi-optimal value $E[c_s]$ in Table 2.

Table 2: Best schedule of SA

SA	*AL*	1	2	3	4	5	6	7	8	9	10	72.52
	BL	0	13	17	33	12	7	36	15	14	0	
	S	0	0	28	0	150	75	93	75	159	185	

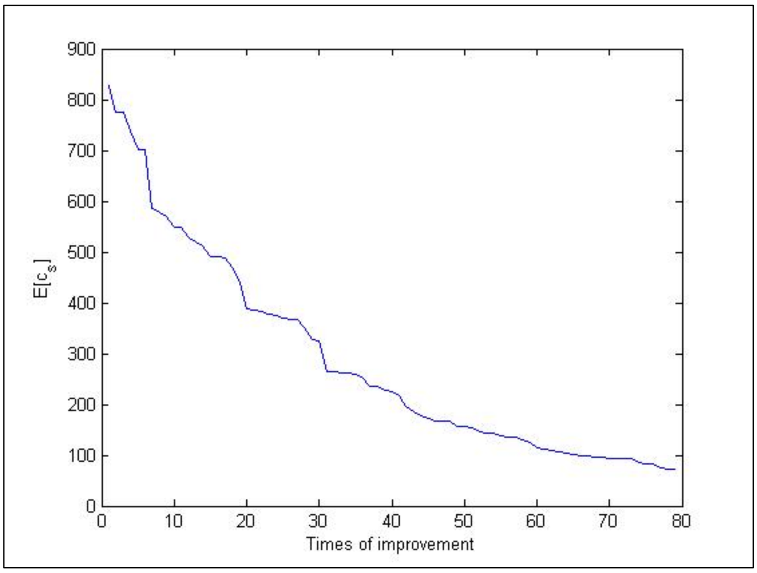

Figure 2: Best solutions during running SA

VII. Conclusions

Many projects are executed in uncertain environment instead of stochastic environment considering the uniqueness of project, where historical data is not enough. In this paper, we explored developing project schedule with solution robustness in uncertain environment. Based on uncertainty theory, we considered logistics project and built an uncertain model for robust project scheduling. Then we made effort to transform the model to a crisp one. Relying on 99-method and SA, we designed an intelligent algorithms. After that, a numerical experiment was used to execute our algorithm. For future works, robust project scheduling coping with disruption caused by uncertain resource availability is interesting and challenging.

Submitted: February 28, 2015 Accepted: November 5, 2015

Acknowledgements

The work is partially supported by the National Natural Science Foundation of China (No.71371141) and the Fundamental Research Funds for the Central Universities.

References

Artigues, C. and Roubellat, F. (2000), A polynomial activity insertion algorithm in a multi-resource schedule with cumulative constraints and multiple modes, European Journal of Operational Research, Vol. 127, No. 2, pp. 197–316.

Bhattacharyya, R., Chatterjee, A., and Kar, S. (2013), Uncertainty theory based multiple objective mean-entropy-skewness stock portfolio selection model with transaction costs, Journal of Uncertainty Analysis and Applications, Vol. 1, No. 1, Article 16.

Blazewicz, J., Lenstra, J. K., and Kan, A. (1983), Scheduling subject to resource constraints: classification and complexity, Discrete Applied Mathematics, Vol. 5, No. 1, pp. 11–24.

Chen, X. (2011), American option pricing formula for uncertain financial market, International Journal of Operations Research, Vol. 8, No. 2, pp. 32–37.

Chtourou, H. and Haouari, M. (2008), A two-stage-priority-rule-based algorithm for robust resource-constrained project scheduling, Computers & Industrial Engineering, Vol. 55, No. 1, pp. 183–194.

Gao, Y. (2012), Uncertain models for single facility location problems on networks, Applied Mathematical Modelling, Vol. 36, No. 6, pp. 2592–2599.

Herroelen, W. and Leus, R. (2004), The construction of stable project baseline schedules, European Journal of Operational Research, Vol. 156, No. 3, pp. 550–565.

Herroelen, W. and Leus, R. (2005), Project scheduling under uncertainty: Survey and research potentials, European Journal of Operational Research, Vol. 165, No. 2, pp. 289–306.

Huang, H. and Ke, H. (2014), Pricing decision problem for substitutable products based on uncertainty theory, Journal of Intelligent Manufacturing, Doi:
10.1007/s10845–014–0991–7.

Ji, X. and Yao, K. (2014), Uncertain project scheduling problem with resource constraints, Journal of Intelligent Manufacturing, Doi: 10.1007/s10845– 014–0980–x.

Ke, H. (2014a), A genetic algorithm-based optimizing approach for project time-cost trade-off with uncertain measure, Journal of Uncertainty Analysis and Applications, Vol. 2, No. 1, Article 8.

Ke, H. (2014b), Uncertain random time-cost trade-off problem, Journal of Uncertainty Analysis and Applications, Vol. 2, No. 1, Article 23.

Ke, H., Liu, H., and Tian, G. (2015), An uncertain random programming model for project scheduling problem, International Journal of Intelligent Systems, Vol. 30, No. 1, pp. 66–79.

Khemakhem, M. A. and Chtourou, H. (2013), Efficient robustness measures for the resource-constrained project scheduling problem, International Journal of Industrial and Systems Engineering, Vol. 14, No. 2, pp. 245–267.

Kolisch, R. and Hartmann, S. (1999), Heuristic Algorithms for the Resource-Constrained Project Scheduling Problem: Classification and Computational Analysis, Springer.

Lambrechts, O., Demeulemeester, E., and Herroelen, W. (2008a), Proactive and reactive strategies for resource-constrained project scheduling with uncertain resource availabilities, Journal of Scheduling, Vol. 11, No. 2, pp. 121–136.

Lambrechts, O., Demeulemeester, E., and Herroelen, W. (2008b). A tabu search procedure for developing robust predictive project schedules. International Journal of Production Economics, Vol. 111, No. 2, pp. 493–508.

Liu, B. (2007), Uncertainty Theory, 2nd Edition, Springer.

Liu, B. (2009). Some research problems in uncertainty theory. Journal of Uncertain Systems, Vol. 3, No. 1, pp. 3–10.

Liu, B. (2010), Uncertainty Theory: A Branch of Mathematics for Modeling Human Uncertainty, Springer.

Mehta, S. V. and Uzsoy, R. M. (1998), Predictable scheduling of a job shop subject to breakdowns, IEEE Transactions on Robotics and Automation, Vol. 14, No. 3, pp. 365–378.

Peng, J. and Yao, K. (2011), A new option pricing model for stocks in uncertainty markets, International Journal of Operations Research, Vol. 8, No. 2, pp. 18–26.

Pritsker, A. A. B., Waiters, L. J., and Wolfe, P. M. (1969), Multiproject scheduling with limited resources: A zero-one programming approach, Management Science, Vol. 16, No. 1, pp. 93–108.

Qin, Z. and Kar, S. (2013), Single-period inventory problem under uncertain environment, Applied Mathematics and Computation, Vol. 219, No. 18, pp. 9630–9638.

Tian, W. and Demeulemeester, E. (2014), Railway scheduling reduces the expected project makespan over roadrunner scheduling in a multi-mode project scheduling environment, Annals of Operations Research, Vol. 213, No. 1, pp. 271–291.

Van de Vonder, S., Demeulemeester, E., and Herroelen, W. (2008), Proactive heuristic procedures for robust project scheduling: An experimental analysis, European Journal of Operational Research, Vol. 189, No. 3, pp. 723–733.

Van de Vonder, S., Demeulemeester, E., Herroelen, W., and Leus, R. (2006), The trade-off between stability and makespan in resource-constrained project scheduling, International Journal of Production Research, Vol. 44, No. 2, pp. 215–236.

Wu, S. D., Storer, R. H., and Pei-Chann, C. (1993), One-machine rescheduling heuristics with efficiency and stability as criteria, Computers & Operations Research, Vol. 20, No. 1, pp. 1–14.

Yang, X. and Gao, J. (2013), Uncertain differential games with application to capitalism, Journal of Uncertainty Analysis and Applications, Vol. 1, No. 1, Article 17.

Yao, K. (2013), Extreme values and integral of solution of uncertain differential equation, Journal of Uncertainty Analysis and Applications, Vol. 1, No. 1, Article 2.

Zhang, B. and Peng, J. (2013), Uncertain programming model for uncertain optimal assignment problem, Applied Mathematical Modelling, Vol. 37, No. 9, pp. 6458–6468.

Zhang, X. and Chen, X. (2012), A new uncertain programming model for project scheduling problem, Information: An International Interdisciplinary Journal, Vol. 15, No. 10, pp. 3901–3910.