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Original article

Modeling and Optimization Algorithms in Ship Weather Routing*

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Abstract

Efficient and sustainable sea transport is a key aspect to ensure cost competitive ship operation. The constant need to increase economic feasibility, energy efficiency and safety while complying with emission regulations motivates further developments and improvements in voyage optimization and weather routing systems. These systems optimize a voyage based on meteorological and oceanographic information taking into account ship characteristics and routing information. The quality of the provided route not only depends on the quality of this data, but also on the modeling of the optimization problem and the algorithm chosen to solve it. Due to the wide range of mathematical approaches and consequently challenges in decision making, this paper aims to give a comprehensive and comparative overview of the existing state-of-the-art methods by a thorough literature review and elaboration of different modeling approaches, optimization algorithms, and their application in weather routing systems. The research shows that approaches range from modeling the weather routing problem as a constrained graph problem, a constrained nonlinear optimization problem or as combination of both. Based on the formulation of the ship weather routing optimization problem different methods are used to solve it ranging from Dijkstra's algorithm, dynamic programing and optimal control methods to isochrone methods or iterative approaches for solving nonlinear optimization problems. However, it can be concluded that the determination whether an approach is suitable, produces sufficient results and may be recommended, strongly depends on the specific requirements concerning optimization objectives, control variables and constraints as well as the implementation.

Keywords: Weather routing, Mathematical modeling, Optimization problems, Voyage optimization, Algorithms

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I. Introduction and Methodology

Optimizing a ship's voyage is one of the main objectives of a shipping company in order to be cost competitive. It is significantly driven by the need to operate a ship as cost efficient, energy efficient and safe as possible during every voyage in its lifetime. Voyage planning is supported by weather routing systems on a large number of vessels nowadays. Meteorological and oceanographic information as well as ship characteristics and routing information provide the basis to optimize each voyage. Depending on the requirements of the ship's operator and the shipping business, the main objective can be to optimize a ship's voyage concerning energy efficiency, voyage duration, safety or combinations of these aspects. In case of a flexible time of arrival, a minimum time or minimum total cost optimization problem needs to be solved. Often, though, a fixed arrival time is obligatory while the objective is to minimize fuel costs. Besides time constraints, further restrictions include ship characteristics, safety considerations and geographic conditions that are mainly routing restrictions due to land, shallow waters, icebergs, mines or traffic separation schemes. Ship characteristics are mostly considered by a hydrodynamic model reflecting the ship's behavior and responses, its speed profile and fuel consumption when facing wind, waves, current and other environmental conditions.

The ship's speed is influenced by its engine power as well as its calm water and added resistance potentially leading to involuntary speed reductions, while voluntary speed reductions aim to increase safety. Thus, constraints may refer to the maximum available engine power or to speed limitations. Besides the ship itself weather conditions have a great impact. They may be characterized as constant or stochastic. The latter are obtained by considering not only forecasted but also analyzed historical data, hereby accounting for possible forecast errors. Constant weather conditions, in contrast, can refer either to the assumption of neither wind nor ocean current, or to the adoption of the forecasted data as true data for each location at the respective time of passing. In addition, safety requirements such as maximum allowed wave heights or critical encounter periods and angles are crucial to avoid critical occurrences such as slamming or parametric rolling.

Considering these constraints and the objective function, the geographical position and time at each waypoint can be optimized implying the integration of route and speed optimization. However, all derived voyage plans are predictions with a quality not only dependent on the accuracy of the ship's hydrodynamic model and the weather forecasts but also on the choice of mathematical model and algorithm. Weather routing problems can be modeled as nonlinear continuous optimization problems or discrete optimization problems by discretizing space and/or time. A wide range of solving methods varying from methods in optimal control theory, and dynamic programming to Dijkstra's algorithm can be used to find local or global best routes for ships. The proposed methods utilize either single-objective or multi-objective optimization. Furthermore, the methods differ as to the number of control variables. Typically, either the ship's heading or its engine power may be varied in order to optimize an objective function (e.g. travel time or fuel consumption), or only variations of one condition are considered. For example, one may assume constant engine power and optimize travel time by finding the series of ship's headings resulting in the most favorable exploitation of present weather conditions.

Consequently, this paper aims to give a comprehensive and exhaustive comparative overview of the existing state-of-the-art methods by thorough literature research and elaboration to support the understanding and choice of an adequate approach. Unless otherwise stated, all optimization problems mentioned in this paper are constrained by time (cannot run backwards), geographic conditions, engine control (power is limited) and safety considerations. In addition, no weather forecast errors are considered in the following. The analysis of weather routing algorithms in this paper includes methods based on calculus of variations, optimal control theory and dynamic programming as well as discrete optimization or grid-based methods. Furthermore, evolutionary methods and other approaches used in weather routing models are taken into account.

II. Calculus of Variations and Dynamic Programming

2.1. Calculus of Variations

Calculus of variations aims to minimize or maximize functionals often expressed as integrals, in order to find extremals, thus to find the arc connecting start and destination in a manner leading to minimum voyage time. The optimization is achieved through variation of the parameters that control the trajectory (e.g. time or velocity). This approach is equivalent to solving the Euler-Lagrange equations numerically. A time-independent approach (Hamilton, 1961) is based on the variation of the ship's course by assuming that the environmental field is static and the ship's speed is time-independent. The calculation of a ship's least-time track is approached by Bijlsma (1975) using calculus of variations, or the special case of optimal control. According to Bijlsma (1975), four necessary conditions from the classical calculus of variations (Euler-Lagrange, Weierstrass, Legendre and Jacobi) need to be satisfied by an arc to provide a relative minimum for the least-time track problem. An absolute minimum, thus an extremal, though, requires the consideration of time fronts consisting of a set of points reachable within a certain time step given by wave data being available every 12 hours or interpolated every 6 hours. The points reachable within one time step, hence, define a grid. Besides wave and current data at the grid points, further data that is assigned initially includes departure time and location, arrival location and the ship's data including speed reduction in waves. For each time step, the points reachable along the extremals but also along the Great Circle Route (GCR) and rhumb line are computed. Due to boundaries, such as land, the destination might not be reachable along any extremal from the start. A suitable new point may be introduced as intermediate starting point for a new extremal (see Figure 1). The method can be applied to minimize fuel consumption by considering speed and heading as control variables. The speed is constrained by a minimum and a maximum. However, the fuel function is often derived from empirical data. Thus, due to inhomogeneity of equations to be solved, Bijlsma (1975) considers the application of approximations to be more accurate in practical cases than application of the same numerical method as for the least-time track.

2.2. 3D Dynamic Programming

In the context of weather routing, Dynamic Programming in a two-dimensional approach has been originally applied by de Wit (1990) and Calvert et al. (1991). The discrete optimization problem aims to minimize fuel consumption, while engine power and propeller revolutions are assumed constant throughout the voyage. As only the route is optimized, the focus will be on the three-dimensional approach in the following. 3D Dynamic Programming describes a Forward Dynamic Programming method where both, the ship's power settings and its heading, are considered to minimize fuel consumption (Shao, Zhou, & Thong, 2012). The resulting discrete optimization problem with one objective function and several constraints is solved deterministically. In general, Forward Dynamic Programming is based on Bellman's principle of optimality (Bellman, 1952) and therefore on the idea that a path is optimal if and only if the choice of the previous path is optimal for any intermediate stage. A stage (see Figure 2) describes a small part of the original problem defined by the common value of a stage variable.

The settings of the control variables (in this case engine power and ship's heading) are assumed constant between two consecutive stages. Therefore, the final solution consists of the optimal choice of settings for the control variables at every stage in total resulting in the optimal path. Every stage is composed of many states, while a state is defined by a location (grid point) and a discretized time. The optimization procedure starts with the calculation of the ship's heading from the initial state to each grid point on the second stage, while any headings violating constraints are neglected. For each heading, the fuel consumption upon arrival at the next stage as well as the time travelled between the two stages is calculated for each possible discretized calm water ship speed; calculations for speeds violating constraints are abandoned.

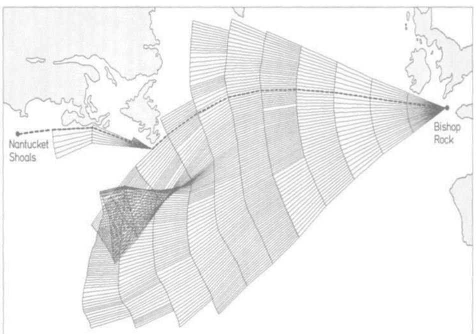

Figure 1: Calculated least-time track
Source: Bijlsma (1975, p. 40)

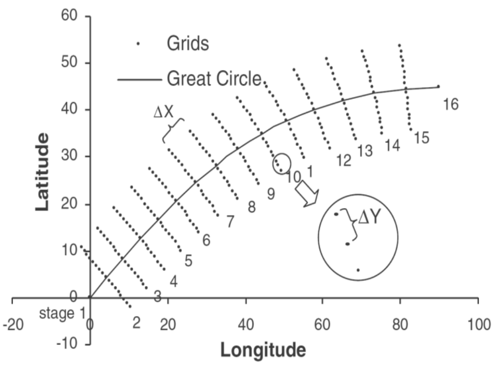

Figure 2: Grid defined by stages perpendicular to the GCR
Source: Shao et al. (2012, p. 244)

In order to enable the comparison of fuel consumed on various routes leading to the same grid point within a certain time interval, all times at a state are defined by the closest smaller discretized arrival time. The fuel consumptions of all routes leading to a certain state (same location and arrival within a predefined time interval) are compared and only the route corresponding to the minimal fuel consumption of each state is saved for future reference. Then the ship's headings from each of the grid points on the second stage to each of the grid points on the third stage are determined and the whole procedure is repeated. Since the optimal route to reach any state is saved for all states of any stage, the final optimal route may be reconstructed easily once the optimal route from the second-last stage to the final state has been determined.

2.3. Iterative Dynamic Programming

Avgouleas (2008) defines weather routing as an optimal control problem with the objective to solve a deterministic nonlinear fuel minimization problem under consideration of safety constraints. A solution is obtained through the development of a MATLAB program employing an Iterative Dynamic Programming (IDP) algorithm (Luus, 2000) based on Bellman's principle of optimality (Bellman, 1952). Using conventional dynamic programming a fine grid is required to ensure convergence to a global optimum increasing computation time (Avgouleas, 2008, p. 66). IDP, therefore, does not take into account a complete grid of feasible states but a single grid point. An initial guess for optimal control of the whole sequence provides the basis for an iterative procedure of piecewise constant control. Both, number of controls and increment can be defined. The number of control settings, thus the number of allowed speeds and headings, allows influencing computation time. Avgouleas considers accurate modeling of the ship's hydrodynamic behavior as important as mathematical modeling and the development of an efficient optimization algorithm. Thus, great attention is paid towards the ship hydrodynamics and wave modeling when aiming "to find the optimal combination of speed and heading to minimize fuel consumption (Avgouleas, 2008, p. 13).

2.4. Isopone Method

The isopone method defines planes of equal fuel consumption (energy fronts) instead of time fronts. As a result, the isopones are not two- but three-dimensional, because they are not only defined by location but also by time (Klompstra, Olsder, & van Brunschot, 1992). A graphical interpretation in three dimensions for the simplified case of uniform weather conditions and no ocean current is provided in Figure 3. Non-uniform weather conditions will not result in barrel-shaped energy fronts but the procedure is similar. The first isopone is determined by calculating the outer boundary of points reachable from the initial point with a fixed amount of fuel when heading roughly along the GCR. Then, all points on the first isopone are regarded as initial points and for each of them, an outer boundary of points reachable with an amount of fuel equivalent to the one considered in the first iteration is calculated. The second isopone is described by the envelope of the resulting energy fronts. Before the third isopone is calculated, several subsectors are defined by parabolic planes parallel to the x_1-x_2-plane as depicted in Figure 3. In each subsector, the point of the boundary, which is closest to the destination, is chosen as an initial point for the next iteration. This procedure is repeated until an isopone reaches the destination, after which the minimum fuel path may be reconstructed by tracing back the headings and speeds used to reach the point on the isopone tangent to the destination. If the final isopone surpasses the destination, it should be recalculated considering a smaller amount of fuel. Alternatively, one may calculate the minimum fuel route from each of the points on the second-last isopone to the destination directly and trace back the one using least fuel. In the case where the last isopone is plane, that is when the estimated time of arrival coincides with the time needed to travel along the minimum fuel path, the resulting path is not only fuel optimal but time optimal as well.

III. Discrete Optimization Methods (Grid-based Approaches)

3.1. Original Isochrone Method

The isochrone method proposed by James (1957) is a practical deterministic method for finding the minimum time route obtained through varying ship headings while assuming constant engine power. It is practical in the sense that the solution is determined by hand. The first isochrone or time front is formed by a line connecting points on the map that a ship departing from the starting port may reach at a specified time by traveling straight ahead in various directions and at constant engine power. Similarly, the next isochrone is calculated as a line connecting points, which may be reached at a specified time by a ship starting from the first isochrone and traveling straight ahead at constant engine power and in a direction perpendicular to the first isochrone. This process may be repeated until an isochrone coincides with the destination and the optimal route can be reconstructed. Implementation of this method for computer applications is problematic since so-called isochrone loops may occur (Szlapczynska & Smierzchalski, 2007, p. 3) .

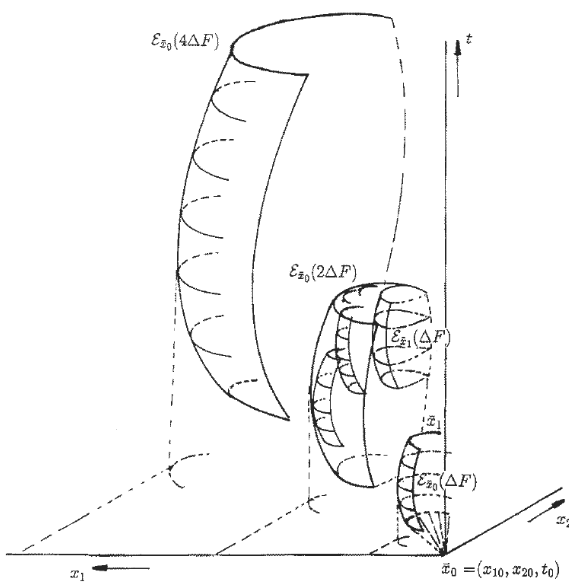

Figure 3: Isopones illustrating the distance a ship can travel within a certain time using a predefined amount of fuel in constant weather conditions
Source: Klompstra et al. (1992, p. 289)

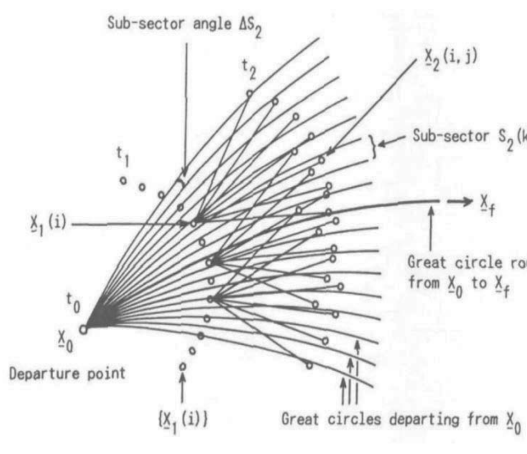

Figure 4: Determination of isochrones using reference routes from start
Source: Hagiwara (1989, p. 21)

3.2. Modified Isochrone Method

Hagiwara (1989) presents a modified isochrone method suitable for computerized calculation since it deterministically solves a discretized optimization problem containing a single-objective function and various constraints regarding time, position (latitude and longitude), control (heading and propeller revolutions) and ship motion (probabilities of slamming, shipping green water, propeller racing, etc.). For this method, the GCR between departure and destination as well as several GCRs departing from the initial port at slightly different angles are used as reference. After the points forming the first isochrone have been calculated as in the basic isochrone method, these points are treated as initial points for the second iteration. Therefore, the points forming the second isochrone are not simply calculated by considering only paths departing perpendicularly with respect to the first isochrone. Instead, all pathways along the GCR leading to the point under consideration of the first isochrone as well as pathways departing at slightly varying angles are considered. In order to maintain a reasonable number of points forming each isochrone, the area around the GCR coinciding with the destination is divided into subsectors defined by the previously described reference routes as depicted in Figure 4. The number of subsections may vary depending on the chosen number of reference routes and the subsector width, which in turn determine the accuracy of the result. Within each subsector, the point furthest along the GCR connecting the departure point and the point under consideration is chosen to be part of the next isochrone. The selected points are then treated as initial points for the next iteration and the same procedure is repeated until the first point on an isochrone coincides with the destination. The optimal path may be found by tracing it back. In addition, an iterative procedure for finding not only a minimum time route but also a minimum fuel or minimum total cost route for a given estimated time of arrival is introduced. As the procedure, though, is based on the assumption of constant engine power, the resulting route does not actually minimize the fuel consumption. The reduced consumption results from the optimal path being the least fuel-consuming path for constant engine power providing on-time arrival. Similarly, a suboptimal minimum total cost route may be found by a cost comparison of the fastest routes found for various values of constant engine power.

3.3. 3D Modified Isochrone Method

According to Lin et al. (2013) this method utilizes a so-called recursive forward technique with a floating grid system, which achieves a route meeting the estimated time of arrival “with minimum fuel consumption and minimum passage time based on the constraints of safety and land avoidance” (Lin, Fang, & Yeung, 2013, p. 184). Variables are the ship’s speed and heading angle. It is “formulated as a multi-stage discrete process subjected to stochastic and dynamic condition” (Lin, Fang, & Yeung, 2013, p. 193). Thus, the floating grid system accounts for the dynamically changing (time) environmental conditions. The stages are defined as the segments between two isochrones and are determined through the ship’s speed in calm water. This also leads to an estimation of the arrival time considering speed and engine power constraints. Each stage consists of several states as also indicated by Hagiwara (1989) in Figure 4. The states are defined by their location taking into account weather conditions, corresponding ship responses and speed as well as passage time and the diversion of the course angle from the GCR, which is chosen as reference route, thus classifying the method as local search one. The speed in this case is influenced by the weather conditions and is subject to voluntary and involuntary speed reduction. Considering the reduced speed, the actual position can be determined recursively. Moreover, the speed is discretized leading to a discrete optimization problem (Lin, Fang, & Yeung, 2013, p. 188). A set of states at a certain time forms an isochrone. At each isochrone, computation time is reduced by choosing the state with the shortest distance along the GCR as new departure point. Consequently, optimal voyage progress and a weight are derived at each stage. The possibility to adjust the weighting factors in the objective functions results in an effective optimization method as to Lin et al. (2013, p. 184). The method allows determining routes of minimum time, minimum fuel consumption or minimum ship motions. Fang and Lin (2015) have developed two routing strategies to further improve the optimization of minimum time routes and minimum fuel routes, ETA (Estimated Time of Arrival) and FUEL.

3.4. Dijkstra's Algorithm

Dijkstra's algorithm for finding the shortest path between two given nodes in a graph with positive edge weights and with more edges than nodes provides a deterministic method for solving a discrete optimization problem consisting of one objective, e.g. minimum distance or time, and only implicitly defined constraints. In weather routing, Sen and Padhy (2015) apply Dijkstra's algorithm for finding the minimum time route in the North-Indian Ocean and Takashima et al. (2009) for determining the minimum fuel route considering variable heading in coastal shipping. Sen and Padhy (2015) assume a 2d 1°x1° grid covering the region under consideration and define the present environmental conditions at each point, and thus in each square, of the grid based on a wave model. The grid provides a reference for a directed graph, which is composed of edges connecting the nodes defined by the midpoints of each square of the grid. For the weather routing problem, Sen and Padhy (2015) define the weights assigned to the edges as the time needed to travel along each edge. The required time is the distance between two nodes divided by the reduced speed. The reduced speed results from the ship's calm water speed for constant engine power as well as involuntary and/or voluntary speed reductions due to the present weather conditions or intentionally decreased speed to avoid potentially dangerous excessive motions. Natural geographical constraints are implicitly defined through the assignment of very large weights. Similarly, high wind velocities or wave heights imply large weights. Engine limits are not considered, because engine power is assumed constant (unless voluntary speed reduction occurs). As to Sen and Padhy (2015), the main disadvantage of the application of Dijkstra's algorithm is that the resulting path is not smooth.

Takashima et al. (2009) address the minimum fuel consumption route problem for coastal merchant ships operating in confined waters by assuming a constant number of propeller revolutions for each optimization run, while varying the heading. The speed is calculated depending on the heading and the weather conditions by applying speed reduction curves.

Computing the ship's resistance and the required propeller thrust the fuel consumption can be derived. The number of propeller revolutions is gradually adjusted in each optimization run in order to meet the required time of arrival. The grid is defined by nodes with a spacing of two miles located on lines perpendicular to a defined standard route. Although this method leads to fuel savings particularly due to avoidance of strong opposite currents in the region of Japan as well as in the case of updated weather forecasts and subsequent rerouting, the obtained route is only a sub-optimal route and not a global optimum. Moreover, Takashima et al. (2009) as well as Sen and Padhy (2015) only deal with single-objective optimization problems (route). The problem of finding the shortest route to a destination that should additionally be reached in the shortest possible time, though, leads to a multi-objective optimization problem (route and voyage time). This multi-objective problem with a defined set of boundary values and set of cost functions is addressed by Böttner (2007, p. 5) by applying "a generalization of Dijkstra's algorithm to the case of two objective functions", which has been proposed by Aneja et al. (1983). The so-called implicit enumeration algorithm is divided in two parts with the first one reducing the network by certain operations and the second one applying "a multiple vectorial labelling scheme" to reduce the computational effort (Aneja, Aggarwal, & Nair, 1983, p. 295).

IV. Evolutionary Algorithms

Evolutionary algorithms may provide good approximate solutions of problems that cannot be solved easily by other methods. As stochastic local search methods, evolutionary algorithms are based on a randomly created initial population and further generations created by mutation, selection or reproduction mechanisms. Each population consists of individuals representing candidate solutions to the problem. A fitness is assigned to every individual describing the quality of the specific solution. Individuals with the highest quality are selected as parents for a new generation. The process of producing generations is terminated when a satisfactory solution has been found. The level of satisfaction that the found solution achieves is highly dependent on the start point (population) and mutation function. Applied in weather routing methods the algorithm aims to solve multi-objective optimization problems, thus optimizing speed and course of the ship throughout its voyage.

4.1. Real-Coded Genetic Algorithm

The Real-Coded Genetic Algorithm proposed by Maki et al. (2011) is an evolutionary calculation technique to solve a multimodal objective function problem. As to Maki et al. (2011), the weather-routing problem can be classified as optimization problem in a continuous domain that can be addressed by two kinds of genetic algorithms, the bit-string and the real-coded genetic algorithm. While the genotype in the bit-string approach is a binary string, the real-coded approach considers real-valued vectors, which is a more efficient approach for strongly intervariable dependencies as in weather routing (Maki, et al., 2011, p. 313). The GCR gives an initial solution, while a real-coded ensemble crossover operator generates new candidate solutions and the set of feasible solutions, thus the next population, is derived by using the Just Generation Gap selection model. As objective functions, three types are considered with “different weight ratios between fuel efficiency and ship safety in parametric rolling” (Maki, et al., 2011, p. 311). By running the optimization method with different weight ratios, an appropriate one can be determined. Control variables are the waypoints’ latitudes and the propeller revolutions. When assuming the GCR as initial solution and choosing appropriate weights for the objective functions the Real-Coded Genetic Algorithm is a robust method.

4.2. Multi-Objective Genetic Algorithm

This procedure introduced by Hinnenthal (2008) aims for the optimization of both, time and fuel consumption by applying a Multi-Objective Genetic Algorithm (MOGA). It stochastically solves a discretized nonlinear optimization problem containing two objective functions and several constraints. Safety constraints include slamming, parametric rolling and vertical as well as lateral accelerations. Since the various constraints imposed by engine limitations as well as safety considerations, force the objective functions into multimodal shapes, i.e. functions with local minima, a deterministic optimization method relying on a convex solution space may or may not yield the optimal result. Therefore, Hinnenthal employs the stochastic method of a genetic algorithm. The SIMPLEX algorithm, though, has been implemented representing a popular deterministic method for solving a nonlinear unconstrained optimization problem. The stochastic method compared to a deterministic method, yet, leads to a tenfold increase of required designs with half of the number being necessary to find the global optimum (Hinnenthal, 2008, p. 113).

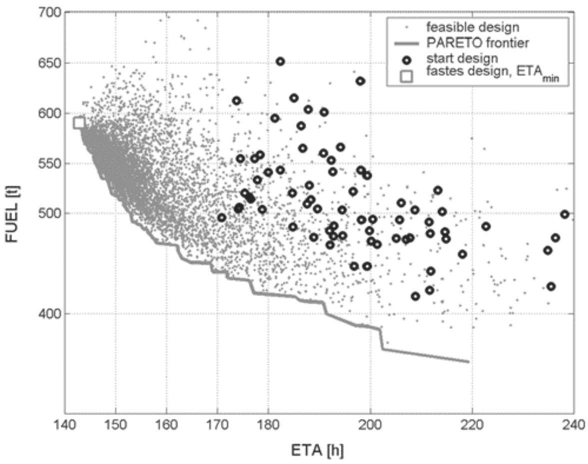

Figure 5: Solution space for the two objectives fuel and ETA including several

Source: Hinnenthal (2008, p. 60)

In order to reduce the computational effort it is essential to reduce the number of designs, hence the number of free variables and objectives, as well as the number of route evaluation points. Therefore, a B-spline technique is applied to model the route including course and speed. The seaway is described by standard spectra and the ship responses are evaluated using linear superposition. The accuracy of the determined routes strongly depends on the quality of the weather forecasts, which is why ensemble forecasts are used. This increases the robustness of the optimized route and allows identifying Pareto optimal routes. As shown in Figure 5, the resulting solution space is bound by the Pareto frontier defined by all solutions for which no further improvement of one of the objectives is possible without impairing the other objective. As a result, not only one optimized route is presented, but the derived set of Pareto optimum routes is provided to the master for decision support.

A Pareto-optimized Multi-Objective Genetic Algorithm is also applied by Marie and Courtielle (2009). Their research focusses on reducing the number of free variables by introducing a “method for spatial and temporal generation of route variants based on a generic and automatic meshing method” (Marie & Courtielle, 2009, p. 140). Here, physical parameters of the geographical environment, meteorological data and ship characteristics are used for discretization. The method “based on spherical rhombus where two of the opposite vertexes are the departure and the arrival points” (Marie & Courtielle, 2009, p. 134) allows automatic meshing and simple adoption to changing route parameters during the voyage. Moreover, Andersson (2015) uses a Pareto Genetic Algorithm with two decision variables (latitude and speed). Here, an initial population based on a grid, roulette wheel selection and mutation are applied. In order to reduce computation time the number of elite sets used to determine domination of individuals is limited by an upper boundary.

4.3. Multi-Objective Evolutionary Algorithm

A Multicriteria Evolutionary Weather Routing Algorithm (MEWRA) utilizing the Strength-Pareto Evolutionary Algorithm (SPEA) and the multicriteria ranking method Fuzzy TOPSIS has been proposed by Szłapczynska and Smierzchalski (2009). The set of objective functions comprises minimum passage time, minimum fuel consumption and minimum voyage risks, while considering dynamically changing weather conditions and constraints, such as landmasses, shallow water or icebergs. The initial population is randomly generated from single-objective minimum time and minimum fuel routes resulting from a modified isochrone method. The SPEA algorithm leading to a Pareto-optimal set of routes derives further generations. The optimization process is terminated when the Pareto set cannot be improved by further evolution. In order to select and recommend one single route from the Pareto set, the multicriteria ranking Fuzzy TOPSIS method applies weights according to defined preferences. The method has been enhanced by taking into account weather hazard avoidance through its application to a motor-driven ship by Krata and Szłapczynska (2012). The improved method uses IMO Circ. 1228 providing guidance to the master for avoiding dangerous situations in adverse weather and sea conditions (IMO, 2007). As a result, dynamically changing constraints regarding wave period, ship’s speed and angle of encounter are handled in the optimization process. Further research has been dedicated to completely customizable optimization criteria and constraints allowing the user to “define as many optimization criteria as needed” (Szlapczynska, 2015, p. 342). Two categories of constraints can be distinguished, namely static and dynamic constraints, thus time-independent ones such as piracy areas and time-dependent ones like maximum wind speed. Although additional criteria and constraints increase computation time, an increase from two to six minutes when considering three instead of two criteria and extending constraints from basic landmasses to piracy areas and wind limits is still reasonable (Szlapczynska, 2015, p. 353).

V. Other Modeling and Optimization Approaches in Ship Weather Routing

5.1. DIRECT Method

The ship weather routing problem can be considered as a difficult global optimization problem. The DIRECT method (Diving Rectangles) introduced first by Jones et al. (1993) is a sampling algorithm created for solving exactly this type of problems. It searches for the global optimum without computing the gradient of the objective function. This can be very useful in the case of expensive or even impossible computation of the gradient of the objective function. The algorithm samples points in the search domain and uses the information obtained for the calculation of the next search step. The DIRECT method uses all the information from the search points to determine if a region of the domain should be subdivided in the current iteration or not. As described in Jones et al. (1993) this algorithm is a kind of Lipschitz optimization method without the Lipschitz constant. It searches for both global and local optima simultaneously and global search is not restricted by local optima. It has therefore fast global convergence and can be set up very easily. Unfortunately, the global convergence leads to a large and exhaustive search over the domain. Despite its weaknesses the benefits of this algorithm for the weather routing problem are investigated further by Larsson and Simonsen (2014). As mentioned in this work, a "conceptual weather routing program has been constructed based on the DIRECT algorithm, showing capabilities of finding great circle route, routing around obstacles such as islands, slowing down/speeding up in order to avoid a time dependent storm, utilizing currents and wind for saving fuel, and the combination of the mentioned." (Larsson & Simonsen, 2014, p. 75) This approach has an initial fast convergence towards an optimal solution but takes a long time to find a local optimal route. The model presented in this work is a very first approach, which introduces the DIRECT method for solving different instances of the ship weather routing problem. Further refinements are needed before it can be benchmarked with the other algorithms used for the same optimization problem.

5.2. Combined Approach

A combined approach for the ship weather routing problem is presented by Weber (1995). Here, the ship weather routing process is described based on a state vector and a control vector, containing state and control variables respectively. The method of optimization applied in this model is to find all feasible control and state vectors in order to enable the calculation of a connection between an initial and an end state with minimal costs controlled by the control vectors. The optimization objective is the minimization of fuel consumption with constraints regarding arrival time, safety and maximum speed. Based on this formulation, the algorithm for solving the problem consists of a combination of two types of optimization problems, namely a combinatorial and a nonlinear one. This combined approach consists of the following steps:

1. Solve a nonlinear optimization problem to determine control variables for given state variables, which will result in state trajectories inside the feasible area. Initially, both gradient based approaches and the Newton method were employed for solving this nonlinear optimization problem. Eventually, a decision in favor of an iterative gradient-based approach has been made since it overcomes the high dependency on the start point that the Newton approach shows.
2. Solve a combinatorial optimization problem to calculate the state variables inside the feasible area for given control variables. Dijkstra's algorithm is used to solve this problem in three dimensions, namely two dimensions in space and time. Furthermore, the graph is divided into stages. In order to smooth discretization effects, a smoothing algorithm is applied after the calculation of the next state by Dijkstra's algorithm.

This approach is robust and provides a converging solution. Therefore, it is worth further consideration and development through tuning of the discretization procedure or usage of state-of-the-art solvers for nonlinear optimization problems.

VI. Discussion and Conclusions

Different modeling approaches, optimization algorithms and their application in weather routing systems have been assessed in this paper. The analysis shows that the weather routing problem is treated as a single-objective or multi-objective optimization problem that can be modelled as a constrained graph problem, a constrained nonlinear optimization problem or as combination of both. Depending on the modelling approach, different methods are used to solve it ranging from Dijkstra's algorithm, dynamic programming and optimal control methods to isochrone methods or iterative approaches for solving nonlinear optimization problems. The named grid-based approaches rather consider a single objective. The approaches, such as Dijkstra's algorithm, are most likely suitable for short (coastal shipping) and shortest route calculations due to a comparably long computation time. Furthermore, the resulting path's improvable smoothness may be considered as a drawback. Dynamic programming "can handle strong nonlinearities and any type of constraint easily. It is less prone to be trapped in a local minimum (or maximum) than a numerical algorithm stemming from the classical calculus of variations (like the steepest descent algorithm)" (Avgouleas, 2008, p. 66). Methods using calculus of variation or optimal control theory, though, are able to find a global minimum (e.g. concerning fuel consumption), but they are also sensitive as to the number of control variables. Likewise, isochrone methods may be limited regarding the number of constraints and variables taken into account. In order to consider multiple objectives, mostly evolutionary approaches are chosen to provide good approximations. However, Hinnenthal states that "a genetic algorithm based method will never reach the computational speed of a deterministic or graph theory based method."

(Hinnenthal, 2008, p. 119) In addition, other approaches such as the DIRECT method or a combined method for solving a combinatorial and a nonlinear optimization problem have been applied. Nevertheless, all applied methods have benefits and deficiencies as indicated in each section. Whether an approach is suitable, produces sufficient results and may be recommended strongly depends on the requirements and the implementation. Thus, the approach needs to be chosen or adapted deliberately according to the specific requirements regarding optimization objectives, control variables and constraints as well as accuracy and computational effort.

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