

Original article

## Two-Stage Supply Chain Network Design Problem with Interval Data\*

*Masoud SANEI<sup>1</sup>, Ali MAHMOODIRAD<sup>2†</sup>, Sadegh NIROOMAND<sup>3</sup>*

<sup>1†</sup> Department of Mathematics, Central Tehran Branch, Islamic Azad University, Tehran, Iran, masoudsanei49@yahoo.com

<sup>2†</sup> Department of Mathematics, Masjed-Soleiman Branch, Islamic Azad University, Masjed-Soleiman Iran,  
alimahmoodirad@yahoo.com, Corresponding Author

<sup>3</sup> Departments of Industrial Engineering, Firouzabad Institute of Higher Education, Fars, Iran, sadegh.niroomand@yahoo.com

### Abstract

Supply chain is usually represented by a network (which is called supply chain network) that contains some nodes. In a supply chain network these nodes are suppliers, plants, distribution centers and customers which are some facilities connected by some arcs to each other. The arcs connect the nodes in the direction of their production flow, meaning that each arc shows a route between the facilities for transporting the products. A multi-stage supply chain network (MSCN) is defined as a sequence of multiple supply chain network stages. This paper addresses a typical supply chain network problem which is based on a two-stage single-product system under uncertain conditions such that both cost and constraint parameters are interval numbers. The combination of these uncertain parameters are considered in this typical problem for the first time. In this case, two different order relations (the order relations  $\leq_{UC}$  and  $\leq_{HW}$ ) for interval numbers are considered. Then, two solution procedures are developed in order relations for the interval two-stage supply chain network design problem. The efficiency of the proposed method is illustrated by a numerical example where it is proved that the relation  $\leq_{HW}$  shows better performance than the relation  $\leq_{UC}$ .

**Keywords:** Supply chain, Interval numbers, Multi-objective.

## I. Introduction

Supply chain is often represented as a network (called a supply chain network) that comprise of some nodes (e.g. suppliers, plants, distribution centers (DC) and customers) representing some facilities plus some arcs. The arcs connect the nodes in the direction of their production flow. A multi-stage supply chain network (MSCN) is defined as a sequence of multiple supply chain network stages. In this case the flow can only be moved between two consecutive stages. Usually, for a connecting arc, the outgoing node and the incoming node are defined as depot and source respectively (Costa et al. 2010; Guo et al. 2015; Sabzehpvar et al. 2015). The MSCN problem involves the choice of some facilities (e.g. plants and DCs) to be established and moreover a distribution network design satisfying the demand of customers with minimum cost.

In recent years, many studies have been done to design different MSCNs. Syarif et al. (2002) considered a multi-source, single-product MSCN design problem which was formulated as a mixed integer linear programming problem. They proposed a spanning-tree based genetic algorithm that applies prufer number representation to solve the problem. Yeh (2006) used a modification of the mathematical model of Syarif et al. (2002) to formulate the same problem. He suggested an efficient hybrid heuristic algorithm that is a combination of a greedy method, linear programming technique and three local search algorithms. Altiparmak et al. (2009) considering a single-source, multi-product MSCN design problem proposed a solution method based on steady-state genetic algorithms with a new encoding structure.

Olivares-Benitez et al. (2013) introduced a supply chain design problem based on two-echelon single-product network. The problem was formulated by a bi-objective mixed integer linear model. A meta-heuristic algorithm that uses concepts of greedy functions, scatter search, path relinking and mathematical programming together, was proposed as the solution method.

In real-world MSCN problems, it is usually difficult to estimate the actual value of parameters (e.g. transportation cost, delivery time, amount of goods delivered, under-used capacity, demands, etc.) as they may be tolerated because of many reasons. Depending on different aspects, those, fluctuate due to uncertainty in judgment, lack of evidence, insufficient information and the like and so on. For example, in MSCN, transportation costs of each stage are affected by the amount of transported goods. However, in the real word applications, we may face some difficulties arising from considering fixed charges which are independent of the amount transported. This reality was a motivation for some recent studies on different MSCN problems in both crisp and uncertain environments.

Petrovic et al. (2001) proposed a heuristic method to calculate the order quantities in a supply chain problem. They applied fuzzy sets to satisfy the assumption of uncertain demands and delivery due-dates. A special purpose simulating tool was suggested by Petrovic (2010) to study performance of supply chains with uncertain parameters. Chen and Lee (2004) formulated a supply chain scheduling problem with multi-product, multi-stage and multi-period properties by a mixed integer non-linear model. Aliev et al. (2007) integrated a multi-period, multi-product fuzzy production and distribution aggregate planning model for supply chains. The integration was done considering a sound trade-off between the filtrated fuzzy customer demand and the profit. The model was

fuzzified and was solved by a genetic algorithm. Pishvaee and Torabi (2010) employed a probabilistic programming approach in order to formulate a closed-loop supply chain network. The objectives were minimization and maximization of the total costs and the total delivery tardiness respectively. Fallah-Tafti et al. (2014) formulated a closed-loop supply chain network by a multi-objective model. They supposed an uncertain environment including inexact cost coefficients and customer demands. They solved the model by an interactive probabilistic method.

Safi and Razmjoo (2013) considered a fixed charge transportation problem in an uncertain environment, where parameters' values are defined by intervals. They used two different order relations of interval numbers. Then, two solution algorithms were introduced to find an optimal solution for the problem.

Baidya and Bera (2014) developed the solid transportation problem with interval environment. They, after converting the interval solid transportation models into its crisp equivalent, applied the weighted Tchebycheff method to solve.

Kristianto et al. (2014) developed a supply chain network by optimizing inventory allocation and transportation routing. They proposed a fuzzy shortest path into two-stage programming in order to find the global optimum solution.

Khalifehzadeh et al. (2015) considered a four-echelon supply chain network design with shortage. They presented a multi-objective mathematical model to minimize the total operating costs of all the supply chain elements and to maximize the reliability of the system. They solved this problem by a comparative particle swarm optimization algorithm.

Mahmoodirad and Sanei (2016) proposed an optimization method based on meta-heuristics algorithms for the design of a multi-stage, multi-product solid supply chain network design problem.

Bahrampour et al. (2016) presented a three-stage multi-product supply chain network model. To tackle the NP-Hard problem, genetic algorithm is used to solve this problem.

Zhang et al. (2016) developed a mixed integer nonlinear programming model to design supply chain. This model involves three major supply chain stages, including procurement, production, and distribution, and their interactions. To solve such a highly constrained, large scale model, they developed an approach based on an artificial bee colony algorithm.

To the best of our knowledge, a few studies use interval modeling approach in MSCN problem which motivated us to perform this new research. The objective of this new study is to develop a two-stage supply chain network problem under uncertain variables and parameters such that both the variable cost and fixed charge of each route and constraint parameters. The uncertainty is shown using interval numbers (despite other existing publications which applied fuzzy numbers and stochastic parameters to deal with uncertainty. See, Alizadeh Afrouzy et al. 2016, Petridis et al. 2015, Pishvaee & Razmi, 2012). In order to solve such problems, two different order relations for interval numbers are used. A solution procedure is also developed for each order relation for interval two-stage supply chain network design problem.

As for the remainder of the paper, Section 2, reviews some basic definitions and arithmetic operations between two interval numbers. Moreover, two different order relations in comparing interval numbers are studied in this section. In section 3, the two stage supply chain network

problem in crisp and interval environments is formulated. The equivalent crisp problem using order relations is presented by Section 4. To explain the method, a numerical example is solved in Section 5. The paper is ended by a conclusion in Section 6.

## II. Basic definitions

In this section, some necessary backgrounds and notions of interval arithmetic which will be used in this paper are reviewed. Throughout the paper, real numbers will be denoted by lower case letters while upper case letters denote closed intervals. Mainly, there are two different representations of an interval such that,

- 1) Representing the interval  $A$  by its lower bound  $a^l$  and upper bound  $a^u$  as follows,

$$A = [a^l, a^u] = \{t \in \mathbb{R} : a^l \leq t \leq a^u\}, \text{ where } a^l \leq a^u \text{ are real numbers.}$$

- 2) An interval number can also be expressed in the form of center and width of the interval as  $A = < a^c, a^w > = \{t \in \mathbb{R} : a^c - a^w \leq t \leq a^c + a^w\}$ , where  $a^c = (a^u + a^l)/2$ , is center of the interval and  $a^w = (a^u - a^l)/2$ , is radius of the interval.

General mathematical operations such as addition, subtraction, division, scalar multiplication, multiplication between two interval numbers, etc. can be defined very easily as mentioned by (Moore, 1979). The addition of intervals and multiplication by a real numbers  $k$  are defined as,

$$\begin{aligned} A + B &= [a^l, a^u] + [b^l, b^u] = [a^l + b^l, a^u + b^u], \quad A + B = < a^c, a^w > + < b^c, b^w > = < a^c + b^c, a^w + b^w > \\ kA &= k[a^l, a^u] = [ka^l, ka^u] \text{ if } k \geq 0, \quad kA = k[a^u, a^l] = [ka^u, ka^l] \text{ if } k < 0 \\ kA &= k < a^c, a^w > = < ka^c, |k|a^w >. \end{aligned}$$

### 2.1. Order relation between intervals

Order relation between intervals is very important issue in the interval analysis. Theoretically, intervals cannot be compared, they can only be partially ordered. However, when intervals are used in practical applications or when a choice has to be made among some alternatives, the comparison of intervals becomes necessary (Alolyan, 2013). There are many different methods for interval comparison proposed in the literature. Ishibuchi and Tanaka (1990) have proposed several definitions of order relation between intervals for the mathematical programming problems. The definitions are as follows.

**Definition 1.** Let  $A = [a^l, a^u]$  and  $B = [b^l, b^u]$  be two intervals, then the order relation  $\leq_{LU}$  is defined as,

$$A \leq_{LU} B \text{ iff } a^l \leq b^l \text{ and } a^u \leq b^u.$$

**Definition 2.** The order relation  $\leq_{CW}$  between the intervals  $A = < a^c, a^w >$  and  $B = < b^c, b^w >$  is defined as,

$$A \leq_{CW} B \text{ iff } a^c \leq b^c \text{ and } a^w \leq b^w.$$

**Definition 3.** Let  $A = [a^l, a^u] = < a^c, a^w >$  and  $B = [b^l, b^u] = < b^c, b^w >$  be two given intervals, then for minimization problems, the order relation  $\leq_{UC}$  is defined by,

$$A \leq_{UC} B \text{ iff } a^u \leq b^u \text{ and } a^c \leq b^c.$$

Since the interval multi-stage supply chain network (IMSCN) is a minimization problem, in the next sections, the order relation UC is used to solve the problem. Hu and Wang (2006) have proposed a method for comparing two intervals. They tried to avoid some difficulties of other existing relations by the following definition.

**Definition 4.** Let  $A = [a^l, a^u] = < a^c, a^w >$  and  $B = [b^l, b^u] = < b^c, b^w >$  be two given intervals. The order relation  $\leq_{HW}$  is defined as follows.

$$A \leq_{HW} B \text{ iff } a^c < b^c \text{ for } a^c \neq b^c, \quad a^w \geq b^w \text{ for } a^c = b^c ..$$

### III. Problem description and mathematical model

The two-stage supply chain network problem consists of plants, DCs and customers. In the first stage, the plants produce and send the products to DCs. Then, the DCs transport the products to the customers. The objective is to minimize the sum of all transportation costs of the supply chain in order to satisfy all capacities and demand requirements. It is assumed that the customer locations with their demand and the DCs and plants locations with their capacities are known as the parameters of the problem. Therefore, this problem can be formulated as a mixed-integer non-linear programming model.

In the following subsection, first MSCN in the crisp environment is introduced then the problem is modelled when the parameters are defined in interval forms.

#### 3.1. Standard representation of the MSCN

The following notations are used to define the mathematical model:

*Set of indices:*

- set of plants ( $i=1,2,\dots,I$ )
- set of DCs ( $j=1,2,\dots,J$ )
- set of customers( $k=1,2,\dots,K$ )

*Parameters:*

- |          |   |
|----------|---|
| $D_i$    | capacity of plant $i$   |
| $E_j$    | capacity of DC $j$  |
| $C_k$    | demand of customer $k$  |
| $c_{ij}$ | cost of transporting one unit of product from plant $i$ to DC $j$               |
| $d_{jk}$ | cost of transporting one unit of product from DC $j$ to customer $k$            |
| $f_{ij}$ | fixed charge for transporting any amount of product from plant $i$ to DC $j$    |
| $g_{jk}$ | fixed charge for transporting any amount of product from DC $j$ to customer $k$ |

*Decision variables:*

- |          |  |
|----------|--|
| $x_{ij}$ | quantity of product shipped from plant $i$ to DC $j$ |
|----------|--|

$y_{jk}$	quantity of product shipped from DC $j$ to customer $k$
$t_{ij}$	binary variable equal to 1 if $x_{ij} > 0$ and equal to 0 otherwise
$z_{jk}$	binary variable equal to 1 if $y_{jk} > 0$ and equal to 0 otherwise

The mathematical model of the problem is:

$$(P1) \text{ Min } Z = \sum_{i=1}^I \sum_{j=1}^J c_{ij} x_{ij} + \sum_{j=1}^J \sum_{k=1}^K d_{jk} y_{jk} + \sum_{i=1}^I \sum_{j=1}^J f_{ij} t_{ij} + \sum_{j=1}^J \sum_{k=1}^K g_{jk} z_{jk} \quad (1)$$

Subject to:

$$\sum_{j=1}^J x_{ij} \leq D_i \quad i = 1, 2, \dots, I \quad (2)$$

$$\sum_{k=1}^K y_{jk} \leq E_j \quad j = 1, 2, \dots, J \quad (3)$$

$$\sum_{j=1}^J y_{jk} \geq C_k \quad k = 1, 2, \dots, K \quad (4)$$

$$\sum_{i=1}^I \sum_{j=1}^J x_{ij} = \sum_{j=1}^J \sum_{k=1}^K y_{jk} \quad (5)$$

$$x_{ij} \geq 0, y_{jk} \geq 0 \quad \forall i, j, k \quad (6)$$

$$t_{ij} \in \{0, 1\} \quad \forall i, j \quad (7)$$

$$z_{jk} \in \{0, 1\} \quad \forall j, k \quad (8)$$

In this model, the objective function presented by Eq. (1) minimizes the total transportation cost of the supply chain network which includes unit based and fixed charge transportation costs of the product from plants to DCs and from DCs to customers. Constraint Eq. (2) is the plant capacity constraint. Eq. (3) is the capacity constraint for DCs while Eq. (4) represents the demand satisfaction constraint for each customer. Eq. (5) assures the product flow conservation between the first and the second stage and Eq. (6) is the non-negative restriction on the decision variables  $x_{ij}$  and  $y_{jk}$ . Finally, Eqs. (7) and (8) define the Boolean variables of the problem.

### 3.2. Interval representation of the MSCN

Let all of the parameters (i.e. variable cost, fixed charge, supply, capacity and demand parameters) be of interval forms, then the interval two-stage supply chain network problem is formulated as follows:

$$(P2) \text{ Min } Z = \sum_{i=1}^I \sum_{j=1}^J [c_{ij}^l, c_{ij}^u] x_{ij} + \sum_{j=1}^J \sum_{k=1}^K [d_{jk}^l, d_{jk}^u] y_{jk} + \sum_{i=1}^I \sum_{j=1}^J [f_{ij}^l, f_{ij}^u] t_{ij} + \sum_{j=1}^J \sum_{k=1}^K [g_{jk}^l, g_{jk}^u] z_{jk} \quad (9)$$

Subject to:

$$\sum_{j=1}^J x_{ij} \leq_1 [D_i^l, D_i^u] \quad i = 1, 2, \dots, I \quad (10)$$

$$\sum_{k=1}^K y_{jk} \leq_1 [E_j^l, E_j^u] \quad j = 1, 2, \dots, J \quad (11)$$

$$\sum_{j=1}^J y_{jk} \geq_l [C_k^l, C_k^u] \quad k = 1, 2, \dots, K \quad (12)$$

$$\sum_{i=1}^I \sum_{j=1}^J x_{ij} = \sum_{j=1}^J \sum_{k=1}^K y_{jk} \quad (13)$$

$$x_{ij} \geq 0, y_{jk} \geq 0 \quad \forall i, j, k \quad (14)$$

$$t_{ij} \in \{0, 1\} \quad \forall i, j \quad (15)$$

$$z_{jk} \in \{0, 1\} \quad \forall j, k \quad (16)$$

where the inequality relations denoted by  $\leq_l$  and  $\geq_l$  are defined as follows (Moore, 1979):

$$x \leq_l [a, b] \equiv \exists z \in [a, b]; x \leq z, \text{ and } x \geq_l [a, b] \equiv \exists z \in [a, b]; x \geq z.$$

Even though, the right hand side parameters in inequalities of the problem (P2) are delivered in interval values, considering that the decision maker needs a deterministic solution which minimizes the total cost of the problem, therefore, to avoid ambiguousness,  $x_{ij}$ 's and  $y_{jk}$ 's are considered to be crisp values.

#### IV. A relevant multi-objective problem

In this section, the order relation between intervals is used to solve the interval two-stage supply chain network problem.

##### 4.1. The order relation $\leq_{UC}$

Since it is desired to obtain an optimal interval value for the problem (2), a bi-criteria objective function (the right bound of the objective interval and its center) is to be minimized.

$$Z^U = \sum_{i=1}^I \sum_{j=1}^J c_{ij}^u x_{ij} + \sum_{j=1}^J \sum_{k=1}^K d_{jk}^u y_{jk} + \sum_{i=1}^I \sum_{j=1}^J f_{ij}^u t_{ij} + \sum_{j=1}^J \sum_{k=1}^K g_{jk}^u z_{jk} \quad (17)$$

$$Z^C = \sum_{i=1}^I \sum_{j=1}^J c_{ij}^c x_{ij} + \sum_{j=1}^J \sum_{k=1}^K d_{jk}^c y_{jk} + \sum_{i=1}^I \sum_{j=1}^J f_{ij}^c t_{ij} + \sum_{j=1}^J \sum_{k=1}^K g_{jk}^c z_{jk} \quad (18)$$

While there is no preference between values parameters within their intervals for the decision maker, it can be shown easily that the best solution to the problem is obtained when the largest feasible region is used. Therefore, the equivalent crisp bi-objective interval two-stage supply chain network problem can be stated as follow.

$$(P3) \text{ Min } (Z^U, Z^C) \quad (19)$$

Subject to

$$\sum_{j=1}^J x_{ij} \leq D_i^u \quad i = 1, 2, \dots, I \quad (20)$$

$$\sum_{k=1}^K y_{jk} \leq E_j^u \quad j = 1, 2, \dots, J \quad (21)$$

$$\sum_{j=1}^J y_{jk} \geq C_k^l \quad k = 1, 2, \dots, K \quad (22)$$

$$\sum_{i=1}^I \sum_{j=1}^J x_{ij} = \sum_{j=1}^J \sum_{k=1}^K y_{jk} \quad (23)$$

$$x_{ij} \geq 0, y_{jk} \geq 0 \quad \forall i, j, k \quad (24)$$

$$t_{ij} \in \{0, 1\} \quad \forall i, j \quad (25)$$

$$z_{jk} \in \{0, 1\} \quad \forall j, k \quad (26)$$

**Definition 5.** Let  $(\bar{x}, \bar{t}, \bar{y}, \bar{z})$  be a feasible solution of problem (P3). If there is no feasible solution  $(x, t, y, z)$ , of problem (P3) such that  $(Z^U, Z^C) < (\bar{Z}^U, \bar{Z}^C)$  then,  $(\bar{x}, \bar{t}, \bar{y}, \bar{z})$  is called a weak Pareto solution of problem (P3).

In the literature there are several approaches which can be used to obtain Pareto optimal solutions for multi-objective programming problems (Steuer, 1986). Here, we treat the problem (P3) as a multi-objective programming problem and the weighted sum method (Steuer, 1986) will be applied to obtain the optimal solution for the problem.

**Theorem 1.** Any optimal solution to the following problem:

$$(P4) \quad \text{Min } w_1 Z^U + w_2 Z^C$$

Subject to (20) - (26).

where  $(w_1, w_2) \geq 0$  and  $w_1 + w_2 = 1$ , is a weak Pareto solution to the problem (P3).

**Proof.** The proof is straightforward (see Steuer, 1986).

The optimal interval for problem (P2) can be calculated by substituting the optimal solution of problem (P4) in objective function of the problem (P2).

#### 4.2. The order relation $\leq_{HW}$

Considering the order relation  $\leq_{HW}$ , two criteria of the objective function of problem (2) as follow.

$$Z^C = \sum_{i=1}^I \sum_{j=1}^J c_{ij}^c x_{ij} + \sum_{j=1}^J \sum_{k=1}^K d_{jk}^c y_{jk} + \sum_{i=1}^I \sum_{j=1}^J f_{ij}^c t_{ij} + \sum_{j=1}^J \sum_{k=1}^K g_{jk}^c z_{jk} \quad (27)$$

$$Z^W = \sum_{i=1}^I \sum_{j=1}^J c_{ij}^w x_{ij} + \sum_{j=1}^J \sum_{k=1}^K d_{jk}^w y_{jk} + \sum_{i=1}^I \sum_{j=1}^J f_{ij}^w t_{ij} + \sum_{j=1}^J \sum_{k=1}^K g_{jk}^w z_{jk} \quad (28)$$

Therefore, the equivalent crisp bi-objective interval two-stage supply chain network problem can be stated as follow.

$$(P5) \quad \text{Min } (Z^C, Z^W) \quad (29)$$

Subject to (20) - (26).

The problem (P5) is a multi-objective problem and the weighted sum method is used to obtain the Pareto solution for the problem. The details of weighted sum method is explained by the following theorem.

**Theorem 2.** The optimal solution to the following problem:

$$(P6) \text{ Min } w_1 Z^C + w_2 Z^W \quad (30)$$

Subject to (20) - (26).

where  $(w_1, w_2) \geq 0$  and  $w_1 + w_2 = 1$ , is a weak Pareto solution to the problem (P5).

**Proof.** The proof is straightforward (see Steuer, 1986).

The optimal interval for problem (P2) can be calculated by substituting the optimal solution of problem (P6) in objective function of the problem (P2)..

## V. Numerical example

As an example, a two-stage supply chain network that has 2 plants, 3 DCs and 3 customers, which are producing a specific type of product, is considered. The particular conditions on producing and sending the product caused in uncertainty and vagueness in the supply, capacity and demand values in plants, DCs and customers, respectively. Former experiences show that these values can be delivered in interval forms. Another company is given the responsibility of shipping the product from plants to DCs and then to the customers. When the product is shipped through each route in each stage, special care is needed on the shipment that is done by a technician and causes a fixed charge for opening each route in each stage. In each stage, also, variable transportation costs are available in interval forms. We are seeking the best strategy of shipment that satisfies all restrictions and minimizes the total cost of transportation, concurrently. The data in the first and second stages are given in Table 1 and Table 2, respectively.

**Table 1: Cost matrix,  $(c_{ij}, f_{ij})$**

	DC1	DC2	DC3	Supply
<b>Plant1</b>	([3,7], [8,20])	([5,8], [50,70])	([8,12], [8,30])	[50,70]
<b>Plant2</b>	([7,20], [7,17])	([18,30], [20,35])	([5,15], [40,65])	[100,200]
<b>Capacity</b>	[20,50]	[70,120]	[60,100]	

Two above-mentioned order relations are applied to solve the problem of the example. If the example is solved by the order relation  $\leq_{UC}$ , the best optimal solution is as follow,  $x_{11} = 50, x_{12} = 20, x_{23} = 80, y_{13} = 50, y_{22} = 20, y_{31} = 40, y_{32} = 40$ , and other variables are equal to zero. Therefore, the optimal interval is [1559, 3330].

**Table 2: Cost matrix,  $(d_{jk}, g_{jk})$** 

	<b>Customer1</b>	<b>Customer 2</b>	<b>Customer 3</b>	<b>Capacity</b>
<b>DC1</b>	([3,4],[10,30])	([7,11],[6,14])	([7,13],[30,50])	[20,50]
<b>DC1</b>	([10,15],[15,40])	([8,14],[13,25])	([10,17],[20,70])	[70,120]
<b>DC1</b>	([2,5],[8,30])	([4,16],[10,40])	([9,18],[30,45])	[60,100]
<b>Demand</b>	[40,80]	[60,90]	[50,100]	

By using the order relation,  $\leq_{HW}$ , the best optimal solution for the given example is as follow,  $x_{11} = 50, x_{12} = 20, x_{23} = 80, y_{12} = 40, y_{13} = 10, y_{22} = 20, y_{31} = 40, y_{33} = 40$ , and other variables are equal to zero. Therefore, the optimal interval is [1785, 2729].

## VI. Conclusion

This paper addressed a typical supply chain network problem which is based on a two-stage single-product system under uncertainty conditions such that both cost and constraint parameters are of interval numbers. The combination of these uncertain parameters are considered in this typical problem for the first time. To compare and order the intervals, two different order relations were employed (the order relations  $\leq_{UC}$  and  $\leq_{HW}$ ). Based on the applied order relations, the solutions could be generated by solving a bi-objective programming problem. In order to obtain a Pareto optimal solution, we used a typical weighted sum approach and finally the optimal interval solution for the interval two-stage supply chain network problem was obtained by a professional software package CPLEX. The efficiency of the proposed method was illustrated by a numerical example where it was proved that the relation  $\leq_{HW}$  shows better performance than the relation  $\leq_{UC}$ .

As future study on this topic, the problem can be extended to a multi-product supply chain network. Instead of using interval data, use of fuzzy numbers may be interesting to reflect the uncertainty of the problem. Also the researchers of this field may be interested to apply meta-heuristic algorithms to solve large size instances of this problem.

*Submitted: March 31, 2016 Accepted: October 25, 2016*

## VII. Acknowledgements

This study was supported by Islamic Azad University, Central Tehran Branch. The first author is grateful for this financial support.

## References

- Aliev, R. A. Fazlollahi, B. Guirimov, B.G. and Aliev, R. R. (2007), Fuzzy-genetic approach to aggregate production-distribution planning in supply chain management, *Information Sciences*, Vol. 177, No. 20, pp. 4241–4255.
- Altiparmak, F. Gen M. Lin, L. and Karaoglan, I, (2009), A steady-state genetic algorithm for multi-product supply chain network design, *Computers & Industrial Engineering*, Vol. 56, pp. 521–537.
- Alolyan, I. (2013), Algorithm for Interval Linear Programming Involving Interval Constraints, Proceedings of the World Congress on Engineering and Computer Science, Vol. I, WCECS, 23-25 October, San Francisco, USA.
- Baidya, A. and Bera, U. K. (2014), An interval valued solid transportation problem with budget constraint in different interval approaches, *Journal of Transportation Security*, Vol. 7, pp. 47–155.
- Chen, C.L. and Lee, W.C. (2004), Multi-objective optimization of multi-echelon supply chain networks with uncertain product demands and prices, *Computers and Chemical Engineering*, Vol. 28, pp. 1131–1144.
- Costa, A. Celano, G. Fichera, S. and Trovato, E. (2010), A new efficient encoding/decoding procedure for the design of a supply chain network with genetic algorithms, *Computers & Industrial Engineering*, Vol. 59, pp. 986–999.
- Fallah-Tafti, A. Sahraeian, R. Tavakkoli-Moghaddam, R. and Moeinipour, M. (2014), An interactive possibilistic programming approach for a multi-objective close-loop supply chain network under uncertainty, *International Journal of Systems Science*, Vol. 45,(3), pp. 283–299.
- Guo, H. Wang, X. and Zhou, S. (2015), A Transportation Problem with Uncertain Costs and Random Supplies, *International Journal of e-Navigation and Maritime Economy*, Vol. 2, pp. 1 – 11.
- Hu, B.Q. and Wang, S. (2006), A novel approach in uncertain programming part i: New arithmetic and order relation for interval numbers. *Journal of Industrial and Management Optimization*, Vol. 2, pp. 351–371.
- Ishibuchi, H. and Tanaka, H. (1990), Multiobjective programming in optimization of the interval objective function, *European Journal of Operational Research*, Vol. 48, pp. 219–225.
- Moore, R.E. (1979), *Methods and Applications of Interval Analysis*. SIAM, Philadelphia.
- Olivares-Benitez, E. Mercado, R. R. and Gonza, J. L. (2013), A metaheuristic algorithm to solve the selection of transportation channels in supply chain design, *International Journal of Production Economics*, Vol. 145, pp. 161–172.
- Pishvaee, M.S. and Torabi, S.A. (2010), A possibilistic programming approach for close-loop supply chain network design under uncertainty, *Fuzzy Sets Systems*, Vol. 161, pp. 2668–2683.
- Petrovic, D. (2001), Simulation of supply chain behavior and performance in an uncertain environment, *International Journal of Production Economics*, Vol. 71, pp. 429–438.
- Petrovic, D. Roy, R. and Petrovic, R. (1999), Supply chain modeling using fuzzy sets, *International Journal of Production Economics*, Vol. 59, pp. 443–453.
- Safi, M. R. and Razmjoo, A. (2013), Solving fixed charge transportation problem with interval parameters, *Applied Mathematical Modelling*, Vol. 37, pp. 8341–8347.
- Sabzehpourvar, M. and Alavi, S. H. (2015), The role of key parameters in public transportation security, *Journal of Transportation Security*, Vol. 8, pp. 37–40.
- Steuer, R.E. (1986), *Multiple Criteria Optimization: Theory, Computation and Application*. John Wiley, New York.
- Syarif, A. Yun, Y. and Gen, M. (2002), Study on multi-stage logistic network: A spanning tree-based genetic algorithm approach. *Computers & Industrial Engineering*, Vol. 43, pp. 299–314.
- Yeh, W. C. (2006), An efficient memetic algorithm for the multi-stage supply chain network problem. *International Journal of Advanced Manufacturing Technology*, Vol. 29, pp. 803–813.

There is no conflict of interest for all authors.