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Azimuth method for ship position in celestial navigation☆

Van-Suong NGUYEN [a*], Nam-Kyun IM [b], Quang-Dan DAO [c]

[a*] Faculty of Navigation, Vietnam Maritime University, Vietnam, nguyenvansuong@vimaru.edu.vn, Corresponding Author

[b] Department of Navigation Science, Mokpo National Maritime University, Korea, namkyun.im@mmu.ac.kr

[c] Faculty of Navigation, Vietnam Maritime University, Vietnam, daoquangdan@vimaru.edu.vn

Abstract

The methods of celestial navigation to fix the ship position in line with the stars are only applied in the twilight time interval when both the celestial bodies and the horizon apppear simultaneously. This means that these methods cannot be used during the night even if the stars are visible. This paper proposes a novel approach which uses the azimuth of the celestial body in order to establish the great circle equation relating the observed body to the ship position when the celestial bodies appear. In addition, the proposed method does not demand the horizon and sextant equipment as with the previous methods. The key advantage which differentiates this method from previous ones is its ability to determine the ship position during the night when the horizon is invisible. Firstly, the vector calculus is applied to find the mathematical equation for the ship position through analyzing the relationship between the ship position and the great-circle azimuth of the observed body. Secondly, the equation system for the ship position is expanded into a standard system in which the input for the proposed mathematical system are the great-circle azimuth and the coordinates of the observed body. Finally, the numerical technique is also proposed to solve the nonlinear system for the ship position. To verify the validation of this proposed method, a numerical experiment is carried out and the results show that it can be applied well in practice.

Keywords: Azimuth method, Great circle, Celestial navigation, Nonlinear systems, Numerical technique.

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☆ This is a revised version presented at the 4rd Ai-MAST held at Mokpo, Korea, December 12-14, 2016.

1. Introduction

The basic theory of the celestial navigation method is the usage of the measurement altitude of celestial bodies associated with the estimated ship position in order to find out the parameters of the line of position equation (LOP), after that, the equation system for ship position is solved by computer software or plotting on the chart. However, the ship position is actually an intersection of two or more circles of an equal altitude (COP). Due to the complexity of computing COP equations, the Taylor series is applied to simplify the equation of the circle of equal altitude by eliminating the high order small elements out of the initial COP equation, and then the line position of the equation is established. It means that the ship position obtained by the LOP equations is not as accurate as the one calculated using COP equations. Recently, with the rapid development of computer science, some methods have been proposed to solve the COP equations specifically for working out the ship position. Chen al et. (2003, p.230) developed the trigonometric technique to solve the simultaneous COP equations using a spherical coordinate system. According to (Andres, 2008), the COP equation was analyzed using the Cartesian coordinate system instead of the spherical coordinate system, and the vector calculus was then applied to form two equations of COP with three unknowns, the technique of vector expansion was proposed to obtain the ship position. (Tsou, 2012) employed the genetic algorithm which mimics the natural process of biological evolution to search the optimal ship position in LOP equations. On the other hand, (Nguyen and Im, 2014) proposed the other solution to obtain the COP equation using the Cartesian coordinate system, after that the SVD math technique was applied using the least square method to find the ship position with more stability than the normal least square method allows.

Although direct methods to solve COP equations have achieved significant success, all of methods need the star's altitude which is measured between the stars and horizon by a sextant to input into the COP equations. It means that these methods are only applied in the twilight time interval when both the stars and the horizon appear simultaneously. During night as the horizon fades, these methods cannot be used. In this research, in order to overcome the limitations and the disadvantages of previous methods, the authors propose a novel idea to fix the ship position while he celestial bodies are visible. The novel method does not require horizon and sextant equipment. Firstly, a new mathematical equation relationship between the ship position and the observed body is proposed and found by analyzing the vector calculus for the great-circle azimuth of the celestial body. Secondly, the equation system for ship positioning is expanded into a standard system in which the input for the proposed mathematical system is the great-circle azimuth of the celestial body. Finally, the numerical method is also proposed to match the nonlinear system for ship positioning.

2. The celestial sphere on Cartesian coordinate system

The basic concept of celestial navigation includes elements of the celestial sphere such as the latitude (φ) and the longitude (λ) of the ship's position and the Declination (δ) and Greenwich Hour Angle of the celestial body (GHA) are viewed using spherical trigonometry. In this research, the above elements are also measured using the Cartesian coordinate system as shown in Figure 1.

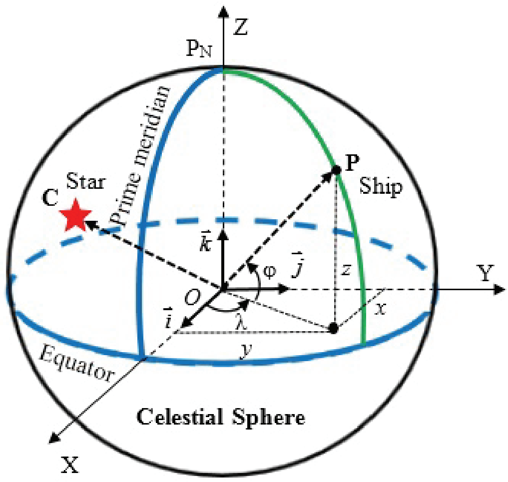

Figure 1: **The celestial sphere on Cartesian coordinate system**

In the Cartesian coordinate system, the radius of the celestial sphere is chosen using $R = 1$ and ($\vec{i}, \vec{j}, \vec{k}$) are the unit vectors. The ship position and celestial body are respectively denoted as $P(X, Y, Z)$ and C(x_C, y_C,

z_C). The formulas shown by (Andres, 2008) are used to express the Cartesian coordinates of ship and celestial body as well as to convert this system into another one. A vector of a ship's position on the sphere is defined by following equation:

$$\overrightarrow{OP} = X\vec{i} + Y\vec{j} + Z\vec{k} \quad (1)$$

Corresponding to the celestial body, the vector has form as follows:

$$\overrightarrow{OC} = x_C\vec{i} + y_C\vec{j} + z_C\vec{k} \quad (2)$$

where, the relation between the Cartesian coordinate and the spherical coordinate of same point is represented as:

$$[X, Y, Z]^T = [\cos\varphi\cos\lambda, \cos\varphi\sin\lambda, \sin\varphi]^T \quad (3)$$

$$[x_C, y_C, z_C]^T = [\cos\delta\cos GHA, \cos\delta\sin GHA, \sin\delta]^T \quad (4)$$

On the contrary, the latitude and longitude of the ship position can be obtained by converting the Cartesian coordinates into spherical ones such as in the following closed form equation:

$$\begin{cases} \varphi = \operatorname{atan}\left(\dfrac{Z^2}{X^2+Y^2}\right) \\ \lambda = \operatorname{atan}\left(\dfrac{Y}{X}\right) \end{cases} \quad (5)$$

Variables and symbols are shown as in Table 1. The dot product and the cross product are also denoted by (.) and (×) symbol respectively.

Table 1: Variables and symbols

Symbol	Variable	Interval
GHA	Greenwich Hour Angle	0 <= GHA <=360° (W to E)
Dec (*δ*)	Declination	-90°(S) <= *Dec* <= +90°(N)
φ	Latitude	-90°(S) <= *φ* <= +90°(N)
λ	Longitude	-180° (W) <= *λ* <=+180° (E)
A	Azimuth	0 <= *A* <=360°

3. The observing azimuth of celestial bodies

At observation time, the use of a compass to measure the azimuth of celestial bodies to obtain values such as (A_1, A_2, A_3...A_n) which are the directional angles from the North Pole to celestial bodies are shown in Figure 2. As known, the light transmits from the stars to observer's eyes in accordance with the shortest arc, thus the light arc from the observer's eyes to the star looks like a great circle. In the sphere, the azimuth angle is created by two great circles, the first one is the meridian arc that goes through the ship position and another one is a great circle that goes through the ship position and star. In this method, the great circle that goes through the ship's position and the star's coordinates is used as Circle of Position (COP) to find out the ship's position.

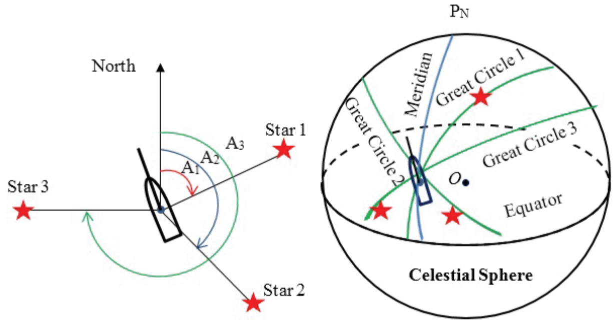

Figure 2: The great-circle azimuth of celestial bodies

The mathematical equation of the great circle in the Cartesian coordinate system proposed by (Earle, 2005) to calculate the shortest route of ship is employed to establish the Circle of position (COP) which passes through both the ship position where the deck officer observes the azimuth of the star and the star's coordinate on the celestial sphere.

The azimuths of the celestial bodies are represented in Figure 2 and Figure 3. The formula for the observing the azimuth of the celestial bodies is constructed by means of the dot product of the two vectors in calculus, expressed by (Earle, 2005) as:

$$\cos A = \left(\frac{\overrightarrow{PV_1}.\overrightarrow{PV_2}}{\left|\overrightarrow{PV_1}\right|.\left|\overrightarrow{PV_2}\right|}\right) \quad (6)$$

where, $\overrightarrow{PV_1}, \overrightarrow{PV_2}$ are respectively the normal vectors of the great circle's plane and the meridian plane at ship position (*P*). By using the theory of vector calculus, these vectors written by (Earle, 2005) can be represented as in Eqs. 7-10.

The normal vector of the great circle at the ship position is determined by a cross product as in the following formula:

$$\overrightarrow{PV_1} = \overrightarrow{OP} \, ? \, \overrightarrow{C} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ X & Y & Z \\ x_C & y_C & z_C \end{vmatrix} \quad (7)$$

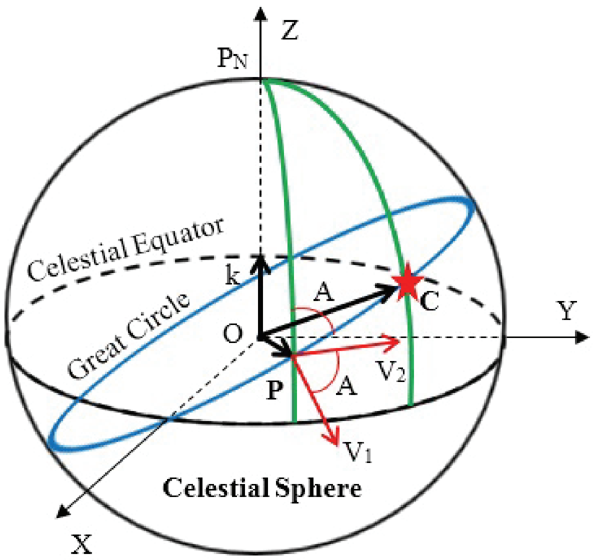

Figure 3: The relationship between the azimuth and the cross product of vectors PV₁, PV₂

In expanding the above formula, we have

$$\overrightarrow{PV_1} = (z_C Y - y_C Z)\vec{i} + (x_C Z - z_C X)\vec{j} + (y_C X - x_C Y)\vec{k} \quad (8)$$

The normal vector of the prime meridian pointing to the ship's position is defined as follows:

$$\overrightarrow{PV_2} = \overrightarrow{OP} \, ? \, \overrightarrow{k} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ X & Y & Z \\ 0 & 0 & 1 \end{vmatrix} \quad (9)$$

Similarly, the final formula of this vector obtained after expanding is shown as follows:

$$\overrightarrow{PV_2} = Y\vec{i} - X\vec{j} \quad (10)$$

By performing steps similar to (Earle, 2005) such as substituting Eq. 8 and Eq. 10 into Eq. 6 and expanding the elements, the formula for observing the azimuth of the celestial body yields as follows:

$$\cos A = \frac{(z_C Y - y_C Z)Y - (x_C Z - z_C X)X}{\sqrt{(X^2+Y^2)\left[\begin{array}{l}(z_C Y - y_C Z)^2 + (x_C Z - z_C X)^2 \\ +(y_C X - x_C Y)^2\end{array}\right]}} \quad (11)$$

Substituting the element (X^2+Y^2) by $(1-Z^2)$, the final mathematical equation of the observed azimuth results as:

$$\cos A = \frac{(z_C Y - y_C Z)Y - (x_C Z - z_C X)X}{\sqrt{(1-Z^2)\left[\begin{array}{l}(z_C Y - y_C Z)^2 + (x_C Z - z_C X)^2 \\ +(y_C X - x_C Y)^2\end{array}\right]}} \quad (12)$$

4. The azimuth method for fixing the ship position in celestial navigation

As the azimuths of three or more stars are measured at a certain time as ($A_1, A_2, A_3,.., A_i$), Eq. 12 is used to calculate the ship position. In this research, we consider the condition of three stars during observation, in other cases, the procedure is carried out similarly. Rewriting equation as follows:

$$f_i(X,Y,Z) = \left(\begin{array}{l}\dfrac{(z_{Ci} Y - y_{Ci} Z)Y - (x_{Ci} Z - z_{Ci} X)X}{\sqrt{(1-Z^2)\left[\begin{array}{l}(z_{Ci} Y - y_{Ci} Z)^2 + (x_{Ci} Z - z_{Ci} X)^2 \\ +(y_{Ci} X - x_{Ci} Y)^2\end{array}\right]}} \\ -\cos A_i\end{array}\right) \quad (13)$$

The nonlinear equation system for determining the ship position with regard to the azimuth of celestial bodies is rewritten in the following form:

$$F(P(k)) = \begin{bmatrix} f_1(X_k, Y_k, Z_k) \\ f_2(X_k, Y_k, Z_k) \\ f_3(X_k, Y_k, Z_k) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (14)$$

The ship position in the Cartesian coordinate system *P(X, Y, Z)*, which is the optimal solution of Eq. 14, is solved by using the numerical method of Newton-Raphson. The detail of the numerical method can be referred to in (Chapra, 2008). After that the solution *P (X, Y, Z)* is converted into the spherical coordinate system by applying Eq. 5 to obtain the ship position $P(\varphi, \lambda)$ with the latitude and longitude.

The iterative process to solve Eq. 14 is carried out as follows: firstly work out the ship position $ES(X_E, Y_E, Z_E)$ just before measuring the azimuths which are chosen as the initial solution *P*(k = 0) of Eq. 14; secondly the initial solution is inserted into Eq. 15 to find the ship position at the next iteration; finally the iterative loops given by Eq. 15 are similarly performed until $F(P(k)) \cong 0$ or $\|P(k+1) - P(k)\|_2 \le \varepsilon$ to yield the optimal position of the ship.

$$P(k+1) = P(k) - J^{-1}(P(k))F(P(k)) \quad (15)$$

Above equation can be rewritten by detail as follows:

where $P(k) = [X_k, Y_k, Z_k]^T$ is the result of the ship position at the *k*-iterative loop, $J^{-1}(P(k))$ is the inverse matrix of the nonlinear system of the ship's position *P*(k). The iterative procedure to find the optimal position of ship can be illustrated as in Figure 4.

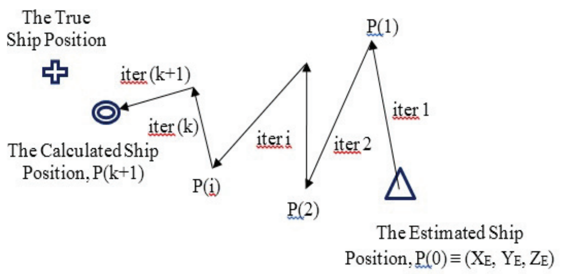

Figure 4: The newton iterative method for solving the nonlinear system

The initial ship position *ES*(X_E, Y_E, Z_E) for the nonlinear system is defined as the last one which deck officer can find out before calculating the ship position at a new observation. The deck officer just needs to input the value of the ship's prior position into the nonlinear system.

By using Eqs. 3-4, the initial ship position along with the latitude and longitude is converted into one using the Cartesian coordinate system with (X_E, Y_E, Z_E). On the other hand, by Eq. 5, the values (X_E, Y_E, Z_E) are also converted into (φ_E, λ_E) easily. In addition, after obtaining the solution of Eq. 14, the ship position in the Cartesian coordinate system (*X, Y, Z*) is transformed to one in the Spherical coordinate system (φ, λ) by applying Eq. 5.

$$\begin{bmatrix} X_{k+1} \\ Y_{k+1} \\ Z_{k+1} \end{bmatrix} = \begin{bmatrix} X_k \\ Y_k \\ Z_k \end{bmatrix} - \begin{bmatrix} \frac{\partial f_1(X_k, Y_k, Z_k)}{\partial X} & \frac{\partial f_1(X_k, Y_k, Z_k)}{\partial Y} & \frac{\partial f_1(X_k, Y_k, Z_k)}{\partial Z} \\ \frac{\partial f_2(X_k, Y_k, Z_k)}{\partial X} & \frac{\partial f_2(X_k, Y_k, Z_k)}{\partial Y} & \frac{\partial f_2(X_k, Y_k, Z_k)}{\partial Z} \\ \frac{\partial f_3(X_k, Y_k, Z_k)}{\partial X} & \frac{\partial f_3(X_k, Y_k, Z_k)}{\partial X} & \frac{\partial f_3(X_k, Y_k, Z_k)}{\partial Z} \end{bmatrix}^{-1} \begin{bmatrix} f_1(X_k, Y_k, Z_k) \\ f_2(X_k, Y_k, Z_k) \\ f_3(X_k, Y_k, Z_k) \end{bmatrix} \quad (16)$$

5. The algorithm of proposed method and numerical experiment for validation

The deck officer observes stars which respectively have the azimuth's magnitude as (A_1, A_2, A_3,... A_i). The times of the identified stars are used to define their spherical coordinates and then the Cartesian coordinates of these stars are found out by using a converted formula. The nonlinear system of working out the ship's position is then conducted by using Newton's iteration method. Beside its main purpose is to fix the ship's position with the stars at night, this method can be also applied to determine the ship's position by both Sun during the day and by stars at twilight. The algorithm of this method is shown as Figure 5.

To verify the validity of this study, a numerical experiment was performed in the gulf of Tonkin of Viet Nam, where three stars were chosen to be observed to acquire data about the ship's position. The calculation process was carried out by Visual Basic 6.0. The detail of observed data from the stars is shown in Table 2.

In area of experiment, the estimated ship position and GMT time were recorded exactly when the stars were observed to obtain their azimuths. The interface of calculation program by Visual Basic 6.0 is represented in Figure 6. The data is entered into the calculation program, and the result obtained is shown as follows.

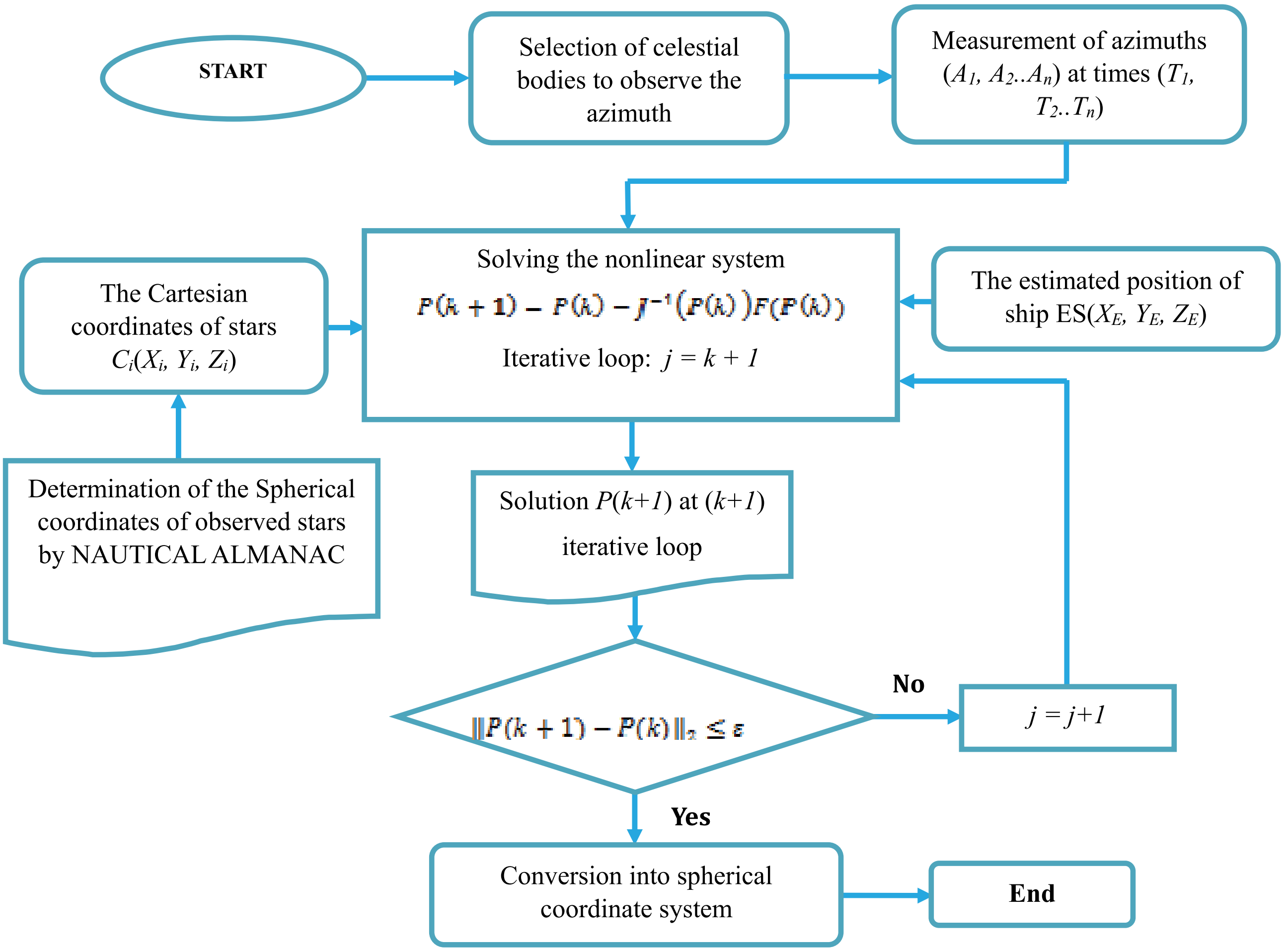

Figure 5: The algorithm of azimuth method for determining ship position

Table 2: The observation data of celestial bodies

Stars	*GHA*	*Dec (δ)*	*x*	*y*	*z*	*Azimuth*
Antares*	$145^0 52'2E$	$26^0 27'7S$	-0,74104	0,50229	-0,4456	140^0
Regulus*	$50^0 34'3E$	$11^0 53'7N$	0,621475	0,755833	0,206119	$89^0 7$
Vega*	$177^0 39'1E$	$38^0 47'9N$	-0,7787	0,031934	0,626581	56^0

GMT (UTC) = $13^h 35^m 28^s$ (June, 9, 2016), True Course = 46 degree, Speed = 10knots, Estimated ship position (Latitude = 20°28’7N; Longitude = 107°13’E).

The result of the experiment’s procedure is synthesized as in Table 3 where the estimated position of the ship is (20°28’7N; 107°13’E) and the true ship position is (20°37’4N; 107°20’1E). The ship position obtained by the proposed method has the respective coordinates 20°35’6N and 107°17’8E.

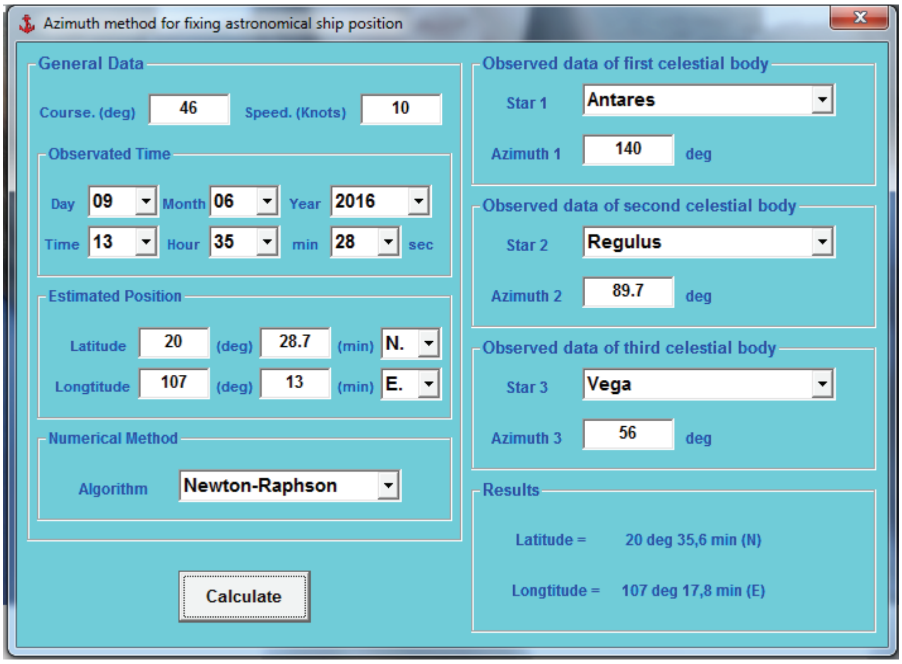

Figure 6: The interface of calculation program for ship position

Table 3: The result of numerical experiment for fixing ship position

Ship Position	Latitude	Longitude
Estimated ship position (Initial position for iterative equation)	20°28’7N	107°13’E
True position of ship on observing celestial bodies	20°37’4N	107°20’1E
Ship position found by the proposed method	20°35’6N	107°17’8E

It is easy to determine the error between the true ship position and the determined one as 2.92NM. This result is not good in comparison to the ship position offered by Global Positioning System, but it is better than the traditional methods currently used in celestial navigation. Particularly, as this experiment to find the ship position is performed at night, our method can be used in this situation. With the previous methods, this work is impossible.

6. Conclusion

In this paper, a new idea in celestial navigation is proposed to fix the astronomical ship position, in which the azimuth of celestial bodies should be applied instead of using the traditional altitude. With this change, the proposed method can overcome the drawbacks of other methods which cannot be used during the night. Due to the use of the azimuth of celestial bodies, the equation system for fixing a ship’s position is quite new in celestial navigation and the solution is applied based on the iterative method in math. Furthermore, the proposed method does not demand sextant equipment. After having the ship’s position figured out by the Cartesian coordinate system, the ship position is determined in spherical coordinate system. The experiment is performed by observing three stars to verify the effectiveness of this approach. The results show that the proposed method can be applied well in practice. In addition, this method can continuously be applied to automatic robotic navigation on other planets where celestial bodies and their azimuth exist.

Submitted : Feb. 1, 2017
Accepted : 25 May, 2017

Acknowledgments

This research was a part of the project titled "Development of Ship-handling and Passenger Evacuation Support System", funded by the Ministry of Oceans and Fisheries, Korea.

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There is no conflict of interest for all authors.